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RESEARCH AND EXPERIMENTAL TESTS IN CONNECTION WITH THE DESIGN OF THE BRIDGE OVER THE DELAWARE RIVER BETWEEN PHILADELPHIA AND CAMDEN.*

BY

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No OTHER modern bridge in this country has been the occasion of so much research and experimentation as the great suspension bridge now under construction over the Delaware River between Philadelphia and Camden. The laying of such a foundation of experiment and test, as a basis for design, was quite usually the procedure when the pioneer long-span metal bridges were built eighty and ninety years ago. Those early engineers were painfully aware of the scantiness of their knowledge and preceded their construction by more or less elaborate tests. In modern engineering there is at times a readiness to accept without any question the rules of design and precedents of construction developed from the "cut and try" methods of our forerunners and to assume no further investigation necessary.

STRENGTH OF WEB PLATES IN COMPRESSION.

One such rule of design which had the sanction of having been handed down from one specification to another since the early days of metal bridge construction is that ascribed to

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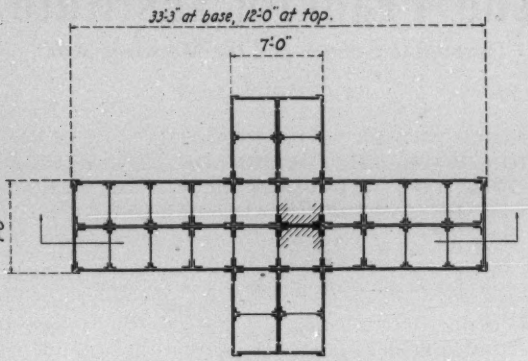
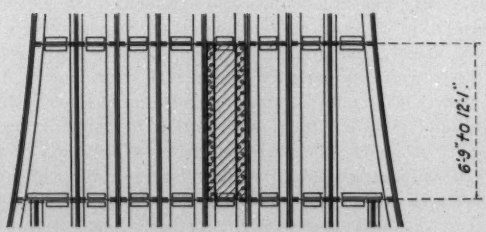
Fairbairn, which limits the width of a web plate in compression to thirty times the plate thickness. In the specifications of the American Railway Engineering Association it is required that "In built compression members * * * the thickness of each



TYPICAL HORIZONTAL SECTION OF TOWER



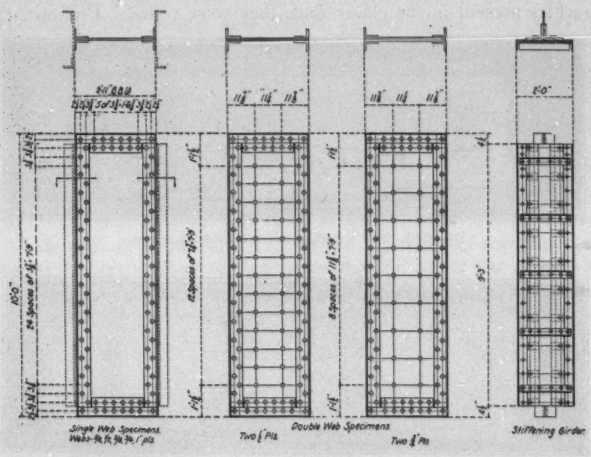
TYPICAL VERTICAL SECTION OF TOWER

Test piece represents shaded portion.

web shall not be less than one-thirtieth of the distance between the lines of rivets connecting it to the flanges."

The towers of the Delaware River Bridge are very large compression members of cellular plate and angle construction. The web plates form a large part of its cross-sectional area and their strength is vital. Tracing back the experimental basis of Fairbairn's rule, it was found to lie in certain small tests made

for Robert Stephenson by Fairbairn and Eaton Hodgkinson at the time that the famous Brittania and Conway bridges were being planned. In 1880 Bouscaren, in this country, reported certain tests of wrought-iron columns for the Cincinnati and Southern Railroad with results which were said to confirm the rule. Investigation of these records showed the evidence to be scant. The Board of Engineers of the Delaware River Bridge Joint Commission therefore undertook a series of compression tests of column web plates, with the coöperation of the U. S. Bureau of Standards.

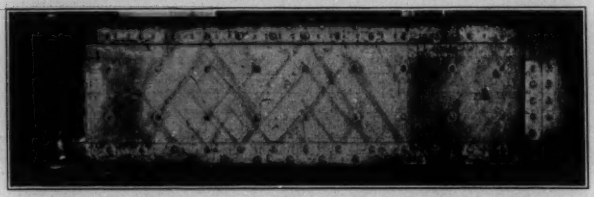


TEST SPECIMENS.

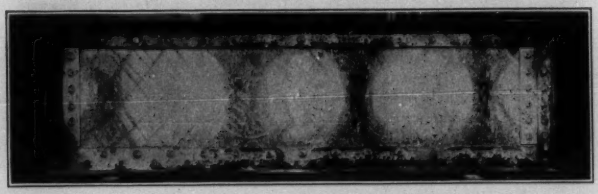
A test section was designed representative of one of the elements—a plate and four angles—making up the cellular construction of the main towers. Lateral support was provided by stiffening girders, intended to simulate the supporting effect of the adjoining sides of the cells in the actual tower sections. The smallest thickness of web planned for the towers was ¾ in. thick, but the test specimens had webs ⅜, ½, ⅝, ¾, 1 and 1½ inches in thickness. The reason for extending the series to a thickness of web so much less than that planned to be used was to increase the value of the information on web buckling which the tests

would furnish. The Board of Engineers was of the opinion that such an investigation once undertaken should contribute to the store of engineering knowledge some new facts in return for the inheritance received from those who have preceded us and that the tests should therefore extend somewhat beyond the limits pertaining strictly to the particular problem in hand.

As anticipated, the characteristic mode of failure of these specimens was by buckling of the webs. In the thinner webs the buckles in all cases developed as exaggerations of the initial buckles present in the plates since they were rolled. Preliminary



Col. G7A. Two ¾-inch webs—south side after test.



Col. G3. One ⅝-inch web—south side after test.

measurements showed some of these initial buckles to be 1/10 inch or so in depth. The thicker web specimens stood up without excessive deformation until the ultimate strength was reached. They then buckled, principally at one point.

During the progress of these tests the effect of the increasing loads was studied, as revealed by the development of what are known as "Hartmann Lueder" lines. These are the lines which appear on the surface wherever the metal has passed its yield point in shear. They were made to show up strongly by lightly coating the specimen with a wash of white Portland cement, which, when dry, flaked off along with the mill scale as the lines

developed. A great deal was learned as to the method of failure of the specimens through this simple procedure, suggested by R. S. Johnston, of the Bureau of Standards.

The local strains in the test specimen were measured by the distant reading carbon pack extensometers developed by the Bureau, while readings were also taken on a 60-inch gauge length by Ames dial compressometers.

The tests indicated that the ultimate strength of the tower columns could be relied on to be between 44,000 and 46,000 pounds per square inch, as these strengths were attained by the specimens with single and double ¾-inch webs. As regards Fairbairn's rule, it was shown to be approximately correct when considered to apply to the clear width of the plate between flange angles. The evidence of these tests is that when applied, as in the A. R. E. A. specification clause, to the width of plate between rivet lines, the rule is over-conservative.

The following table gives the ratio of width of web plates to thickness for the various types of specimens, considering the width as (*a*) unsupported width between toes of flange angles, which was 23 inches and (*b*) as the distance between lines of rivets connecting the web plate to the flanges, which was 26 inches:

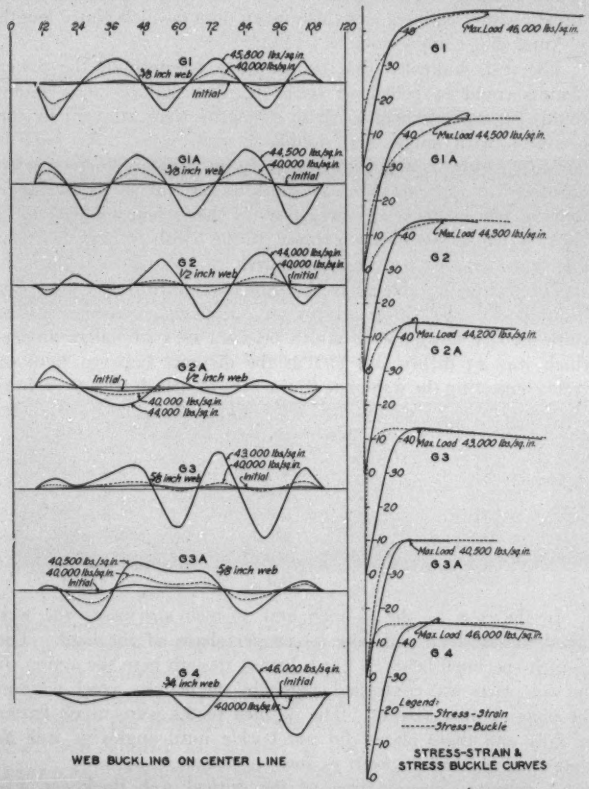
Thickness Web Plate.	Ratio Width to Thickness.	
	(*a*) $\frac{23}{T}$	(*b*) $\frac{26}{T}$
⅜ inch	61.3	69.3
½ inch	46.0	52.0
⅝ inch	36.8	41.5
¾ inch	30.7	34.7
1 inch	23.0	26.0
1½ inches	15.3	17.3

In the case of the ⅜-inch and ½-inch specimens the web failed to stand up and take its proper share of the load. The ⅝-inch specimen behaved much better, though here the action of the web plates was controlled largely by the buckles which were in the plates before testing. The ¾-inch plates were much flatter initially and these plates did not buckle until angles as well as plates had reached the limit of their resistance.

A definite determination of the critical web thickness was prevented by the varying condition of flatness of the plates as well as the fact that the thin plates had, generally, higher elastic strength than the thick plates.

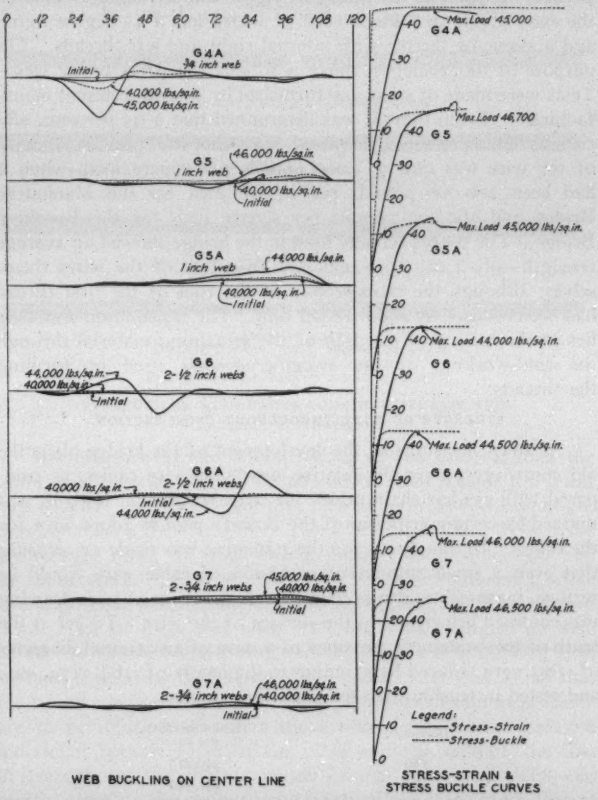
TESTS FOR DESIGN OF CABLES.

During the same time that the web plate investigations were being carried on, a great deal of attention was being given to the study of questions concerning the cables. Having adopted a cable



cross-section with an area twice that of its largest predecessor, it was felt essential that every doubtful point should be cleared up, so far as possible. It has been characteristic of the attitude of the

Board of Engineers throughout the design of the bridge to consider the best example of previous work, eliminate whatever could be shown to be non-essential or mere following of precedent, analyze those features which seemed to require more attention and



WEB BUCKLING ON CENTER LINE

STRESS-STRAIN & STRESS BUCKLE CURVES

carry on such tests as seemed desirable to clear up doubtful points. The result has been the adoption of improvements affecting many of the elements of suspension bridge construction.

WIRE SPLICE.

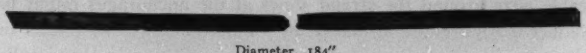
In taking up the design of the cables, one of the first matters considered was the wire splice. The same type of splice has been used in all the wire cable suspension bridges since the Brooklyn Bridge. It consists essentially of right- and left-hand threads on the ends of the two wires, each of which has a 45-degree mitre, and a sleeve or ferrule with internal right and left threads. The purpose of the 45-degree mitre is to lock the wires in the splice. Tests were made of splices as furnished by several different manufacturers. From these it was determined that a 95 per cent. efficiency could be secured, despite the fact that the specified strength of the wire was now 215,000 pounds per square inch, when it had been 200,000 pounds per square inch for the Manhattan Bridge and 160,000 pounds per square inch for the Brooklyn Bridge. The splices actually used in the bridge showed an average strength only 2 or 3 per cent. less than that of the wires themselves, although the cross-section at the root of the first thread had been reduced by about 18 per cent. The explanation doubtless lies in the raising of strength of the remaining material through the cold-working of the swaging process used in forming the threads.

STRENGTH OF WIRE THROUGHOUT CROSS-SECTION.

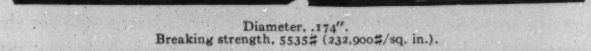
At an earlier stage of the development of the bridge plans the old controversy over the relative merits of wire cables, as compared with eye-bar chain cables for large suspension bridges, was revived by certain criticism of the Board's plan to adopt wire for the cables. In this discussion the statement was made *ex cathedra* that even a small amount of corrosion of cable wire would be serious, inasmuch as it was claimed that the effect of wire drawing was confined principally to the surface of the wire. To get at the truth of this statement, portions of a wire of an original diameter of .189 were reduced by grinding to diameters of .184, .174, etc., and tested in tension with the following results:

Diameter.	Tensile Strength, lb. per sq. in.
.189	229,215
.184	231,300
.174	232,900
.149	236,600
.123	233,700
.099	229,500

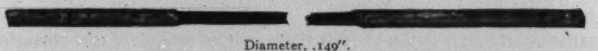
These results would seem to prove that the strength of the wire is quite uniformly distributed throughout the cross-section.



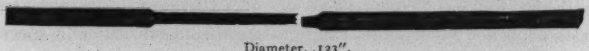
Diameter, .184".
Breaking strength, 6150# (231,300#/sq. in.).



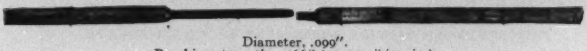
Diameter, .174".
Breaking strength, 5535# (232,900#/sq. in.).



Diameter, .149".
Breaking strength, 4126# (236,600#/sq. in.).



Diameter, .123".
Breaking strength, 2776# (233,700#/sq. in.).



Diameter, .099".
Breaking strength, 1766# (229,500#/sq. in.).
Original wire, Diameter = .189".
Breaking strength, 6429.6# (229,215#/sq. in.).

STRENGTH OF WIRE UNDER COMBINED TENSION AND LATERAL COMPRESSION.

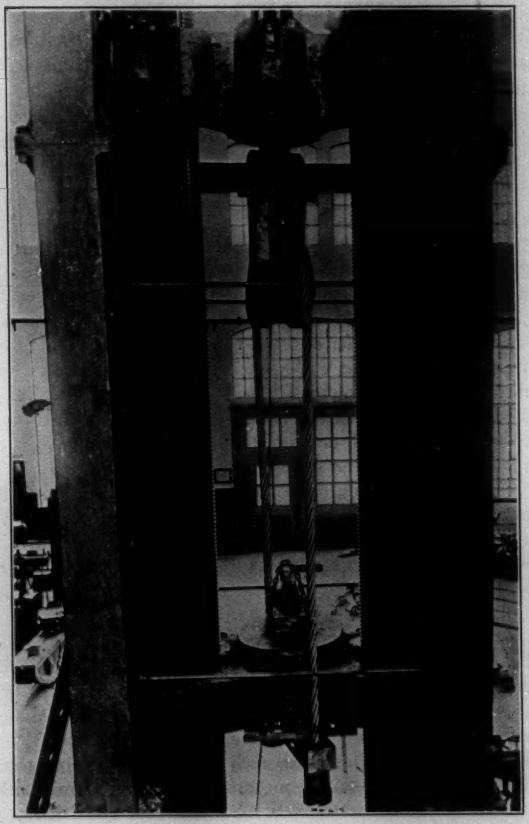
The pressure of the bottom wires of the cable against the tower saddle varies from 1800 pounds to 2800 pounds per lineal inch of wire. It was thought that such a compressive stress might possibly reduce the strength of the wire in tension. Tests were accordingly made of single wires under combined tension and lateral pressure. The tension in the test wire was measured, while lateral pressure of a definite amount was applied to it through two jaws. One of these was lined with two 1-inch lengths of wire parallel to the wire under test, and the other was a finished steel surface, simulating that of the cable saddles. It was found that for lateral compressive loads of 4300 pounds per lineal inch, which are 50 per cent. greater than those which would occur between individual bottom wires of the cable and the saddles, the loss in strength was negligible, a matter of 1 or 2 per cent. This and several other of the preliminary investigations were carried on at the laboratories of the John A. Roebling's Sons Company, at Trenton, N. J., under special arrangements made with that company by the Joint Commission.

STRENGTH OF WIRE IN COMBINED TENSION AND BENDING.

A question naturally arose as to whether the bending of the cable wire around the strand shoes at the anchorages might not reduce their strength at that point. This was tested by comparing the strength of wires pulled in straight tension with those pulled while bent around an 18-inch sheave. The average ultimate strength of the first three wires tested straight was 6151 pounds against an average of 6145 pounds for two wires tested on the sheave. Even the slight decrease shown in these tests was not found in a later series of ten wires where the averages were the same to the last pound. All these wires broke in the straight portions rather than in the portion which was bent and in contact with the sheave. This tendency was also observed in the two later tests of full-sized strands, where, after the failure of both specimens, it was found that practically all wires had broken at points between the strand shoes rather than on them.

STRENGTH OF WIRE ROPES BENT AROUND A SHEAVE.

Going aside for a minute from the investigations bearing on the main cables, a question rather similar to that of the strength of the wire around the strand shoe arose concerning the strength of wire rope suspenders when bent around the cable or cable bands. Expert opinion and experience indicated that there would be a falling off of strength in this case, largely because of the internal cutting action of individual wires against one another in the bent section of the rope. There was a difference of opinion among wire rope manufacturers, however, as to the amount of such loss of strength to be expected in "bridge" rope. "Bridge" rope, it may be explained, is made with a wire rope centre in place of the hemp centre usual for wire rope. One expert thought the loss would be 40 to 50 per cent. and another 15 per cent. No tests had been made previously, except on hemp-centred wire rope. From a study of such tests it was found that the method of strand and rope construction would have a large influence. In the bridge the suspender ropes, 2¼ inches in diameter, are looped around the cable bands, giving a bend with a radius of about 16 inches. Accordingly the specifications were drawn so as to require a certain number of full-sized tests to be made with the rope in tension over a 32-inch sheave. The average results of all



Test of 2¼" suspender rope over 32" sheave.

tests made in this way showed a loss of strength of 11 per cent. from that of the rope in simple, direct tension.

STRENGTH OF WIRE IN STRANDS.

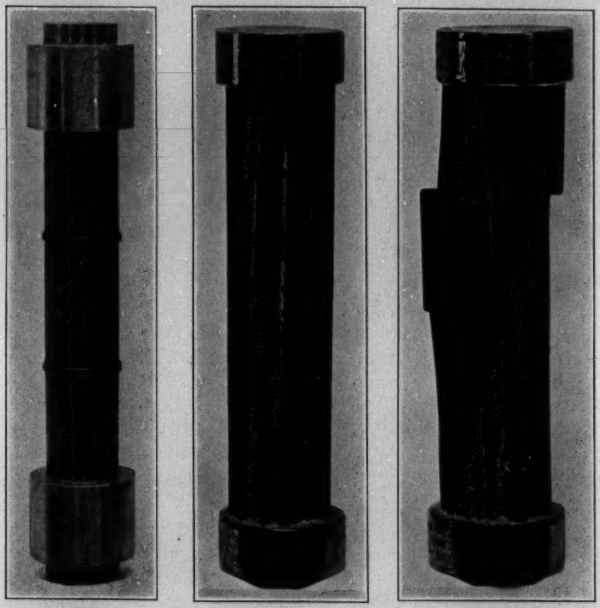
With all the dependence placed on the two 30-inch cables, each an aggregation of 18,666 individual wires, it was naturally a matter of great concern to ascertain as far as possible their strength as a unit. There could be no question of the strength of the single wires, for each 3000-foot length of wire as drawn was tested for tensile strength at each end. The doubt was solely as to the efficiency of the large number in common action. Tests of a parallel wire strand, made thirty-five years ago by George S. Morison for his proposed Hudson River Bridge, had shown only 80 per cent. of the combined strength of the individual wires. Next to the impossible task of testing a 30-inch cable to destruction, the best that could be done was to test strands composed of various numbers of wires and, by noting the efficiency of each, determine whether there was any change in this efficiency as the number of wires was increased.

The first strand tested was at the Roebling works. It contained thirty-seven wires and had an ultimate strength 96 per cent. as great as the sum of the strengths of the individual wires. This was termed an efficiency of 96 per cent. The second and third strands were made and tested at the American Steel and Wire Company works, Worcester, Mass. Each had eighty wires, but in one of the specimens every wire was spliced. The strand without splices developed 95 per cent. efficiency, and, astonishingly enough, the one with all wires spliced was slightly stronger. As the individual splices ordinarily do not quite develop the strength of the wires (the specification requirement being a 95 per cent. efficiency) this result was rather freakish. Evidently the actual pieces of wire in the specimen were somewhat stronger than the individual test pieces on which the efficiency was based.

The third and fourth strands each contained 306 wires, the full number in one of the sixty-one strands of the 30-inch cables. The tests of this size strand were made after the cable design was well advanced. The ends of the previous specimens had been socketed in zinc, according to standard practice with wire rope. The ends of the 306-wire-strand specimens were looped around cast-steel strand shoes of the type used for the connection to the

anchorage eye-bars in the structure. The grooves around the strand shoes were unfinished surfaces of the casting in the test. In the actual work these grooves were machined in order to secure greater precision in placing the wires and better bearing for them.

The full-sized strands showed an efficiency of 94 per cent. This slight falling off from the results of the first tests may



Test No. 1.
37 No. 6 wires.

Test No. 2.
80 No. 6 wires, without splices.

Test No. 3.
80 No. 6 wires, each spliced.

Preliminary tests—parallel wire strands.

mean nothing more than a measure of the greater difficulty in securing uniform tension in the wires while building the larger specimens. The best results in the specimens are probably not as good as the average results in the actual cable-making. In the field work, the tension of the wires is regulated by their sag, which is easily made the same as the guide wires. When the wires have the same sag they have automatically the same tension. In build-

ing up the large specimens every wire had to be pulled to the required tension, measured by a spring balance, as it was wrapped around the strand shoes. The strand shoes were mounted on the supporting strut in such a manner that one shoe could be

Cable strand specimen ready for test.

moved small distances away from the other to compensate for the compression of the strut during the fabrication of the specimen. When all the wires had been placed, the specimen was transferred from this strut to a shipping strut, which served to maintain the initial tension in the wires by holding the strand shoes the correct

distance apart. It is easy to see what a delicate operation the construction of such a specimen would be.

FLEXURE OF BRIDGE CABLES.

Other phases of the cable problem requiring investigation were its behavior under deformation by bending (in which was involved the whole question of secondary stresses), its size after consolidation and the resistance of the cable bands to sliding along the cable.

For the first of these problems a model section of cable previously constructed by the Roebling Company was utilized by the Joint Commission.

This specimen had been built to demonstrate the practicability of flat band seizings for the strands instead of the wire hitherto used. The wire seizings had on past jobs been removed after the strand had been placed in its proper position in the cable. This was not easily done, and had the further disadvantage of permitting the wires in the strand to get out of place subsequently during the compacting operation. A flat band of strip steel was suggested by Holton D. Robinson, and a trial of it was made by using seizings of this type to make up seven bundles of wire, ten feet long, each with approximately the same number of wires as a strand of bridge cable. These seven strands were then compacted by squeezing with clamps so as to produce a ten-foot section of cable 9.4 inches in diameter. The seizings served their purpose well and were later specified for use on the Delaware River Bridge cables.

For the cable bending test this 9.4-inch section was mounted as a beam in the testing machine with cast trunnion bands at the two ends and another casting clamped in the centre where the load was to be applied. Between these bands the cable was completely wrapped with short sections of wire seizing, put on and tightened by hand. To the specimens were attached Ames and Wissler dials so that complete measurements could be made of the deflections during test. The load was applied in small increments, and at times removed, and then reapplied to determine the extent of elastic recovery and of set.

A comparison was made between the deflection of this cable section and that calculated for a similar solid beam of the same cross-sectional area and distribution of voids. For the lightest

loads the cable was almost as stiff as such a beam would be, but as the loads increased the deflections increased much more rapidly. Under a load of 9000 pounds the deflection was some forty-five times as great as that of the equivalent beam. This first test definitely established the general mode of bending and relative flexibility of this type of cable. However, it was found that the constraint from the cast clamps affected the deflection in such a way as to render quantitative interpretation of the results very difficult. The uncertainty as to the tension in the hand-wrapped seizings also prevented the calculation of values for the internal friction developed between wires of the cable.

Arrangement was made to have the specimen re-wrapped mechanically so that the serving would have a definite tension throughout. The trunnion and centre clamps were also loosened so that they put no restraint on the cable. This specimen proved rather stiffer than the other, but even so, when loaded with 22,000 pounds it deflected fifteen times as much as the theoretical beam would have done.

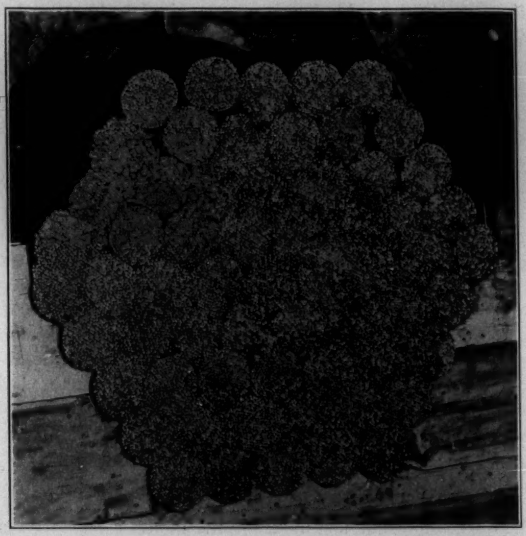
The relative flexibility is, however, no direct measure of the magnitude of the secondary stress caused by the bending. Although slip takes place whenever the longitudinal shear exceeds the friction between wires, and thus permits large deflection of the cable, yet a resisting moment is set up which brings the deflection to a stop again after each increment of load. A rational analysis of this problem was developed by Allston Dana, Assistant Engineer of Design, which enabled the stresses to be calculated from the test data. In turn this permitted an estimate to be made of the secondary stresses in the 30-inch main cables.

FRICTIONAL RESISTANCE OF CABLE BANDS.

Friction was also the heart of another of the cable problems, that of insuring against slip of cable bands down the steep slope of the cables near the towers. The attention given this matter in our work was not from merely theoretical considerations. The bands in other large suspension bridges have frequently given trouble by slipping in this way.

The first experiment was made with the aid of the same 9.4-inch cable section that figured in the bending tests. It was set up on end in the testing machine, a two-part band casting clamped around it, and measurements made of the load required to slip

this band along the cable. Difficulty was encountered in getting quantitative data, as the set-up did not permit of an accurate determination of the tension in the clamping bolts. The readings indicated that the resistance to sliding was fully 60 per cent. of the compacting force. This is too great a percentage to be accounted for by friction alone. The bulging of the cable outside



Model section of cable before compacting.

the limits of the band was undoubtedly a large factor in resisting slip.

The problem was next approached from another angle. If bulging should be found not to play so prominent a part in the behavior of the bands on the 30-inch cables, reliance could at least be placed on the friction of cast-steel bands on galvanized wires. Values for this friction coefficient were sought with small-scale samples. An apparatus was rigged which would permit blocks of steel one inch in length and loaded with any desired amount to be slid along pieces of galvanized wire a few inches in length. The steel blocks were finished with transverse tool cuts, 16 to the

inch, 30 to the inch and smooth. The coefficient of friction was 25 per cent. for the roughest finish and less for the others. The biting of the ridges of the machined surface into the two- or three-thousandths of an inch of zinc galvanizing on the wire was evidently an important factor in the frictional resistance. As a result of what was learned from these tests, the finish of the inside of the bridge cable bands was required to be of a definite degree of roughness, corresponding to that of the specimen with sixteen cuts to the inch. This was an unusual requirement in machining and was successfully met by the shop, although it meant that an extra cut had to be taken. The cable bands were designed on the assumption of a 15 per cent. coefficient of friction being developed.

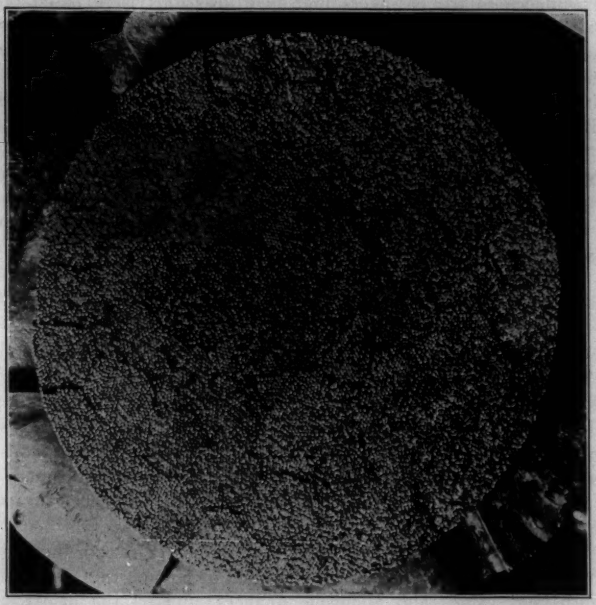
The final test for resistance to slipping was made on the full-scale section of main cable five feet in length. A cable band, finished internally as specified, was clamped on to this section, using the regular heat-treated, high-tensile bolts made for the structure. The test section was stood on end in the large testing machine at Lehigh University and a casting slipped over one end so as to bear on the end of the band. The load was applied in increasing amounts through this ring until slip of the band was detected. This required a load of 190,000 pounds, which amounted again to about 60 per cent. of the gripping force on the band. It is evident that bulging plays a large part in the resistance to slipping, even in the 30-inch cable.

DIAMETER OF COMPACTED CABLE.

Another important use made of the model cable—its reason for being, in fact—was to determine the size of the compacted cable so that the bands could be bored to correct diameter. Preliminary calculations had been made on the basis of the knowledge acquired in the construction of the cables of previous suspension bridges. These figures had been checked in the light of the results of a test conducted by the Roebling Company on the 9.4-inch diameter model and the probable diameter of big cables estimated to be 29-15/16 inches.

The full-sized model was made in close simulation of the conditions that would obtain in the actual cables. The individual wires were taken at random from wire made on a production basis for the bridge. The same number of wires was spliced as

would be spliced in the same length of cable. The strands were each made up separately, with flat band seizings of accepted pattern, and laid up so as to form a hexagonal prism. One of the compacting devices, intended later for the field, was used to squeeze the cable to a cylindrical form, with the wires in tight contact. Each of the six jacks mounted in the hexagonal frame



Model section of compacted cable.

of the squeezer exerted a force of 30 tons, giving far more power than had been available on the cables of the older bridges. The result was a very tight compaction. The minimum diameter measured was 29-15/16 inches.

These and other minor but very interesting investigations represent the experimental and research work carried on up to date in the development of the bridge designs. Such research is of necessity highly practical and may seem as rough to the

research student in physics as it would seem refined to an earlier generation of bridge builders. The tests were in most cases suggested by the questions that arose during the development of the designs and answers were needed to the point and without delay. The phases of the work which were investigated were vital to its success. The knowledge secured from these tests gave confidence to the designers that their decisions were based on something more than precedent and theory. That it was possible to carry out such a program was due to the broad view taken by the Delaware River Bridge Joint Commission, in granting authority for the necessary expenditures, on the recommendation of the Board of Engineers.

THE TOWERS, CABLES AND STIFFENING TRUSSES OF THE BRIDGE OVER THE DELAWARE RIVER BETWEEN PHILADELPHIA AND CAMDEN.

BY

LEON S. MOISSEIFF, C.E., MEM. AM. SOC. C.E.

Engineer of Design, Delaware River Bridge Joint Commission.

THE paper I have the honor to present to you to-night is on the towers, cables and stiffening trusses of the bridge now being built over the Delaware River between Philadelphia and Camden. The subject extends over so large a field, it covers so many items of engineering interest, each one deserving its own detailed treatment, that I will of necessity confine myself to the main considerations governing the design of the structure and only touch on its more salient features.

The planning of a municipal bridge is the art of coördinating into a well-balanced system the demands of commerce and travel with land values and allowable finances, and of the available materials and means of construction with economic efficiency and monumental beauty. Some of these factors are determined by laws, some are directed by public opinion, and others are left to the knowledge and judgment of the engineers and architects. Thus, for the Delaware River Bridge the navigation requirements of the Port of Philadelphia obviously demanded a free river between pier-head lines and a structure tall enough to pass at all times the ships that ply over the high seas. This found its legal expression in the permit of the War Department for a clear span

between pier-head lines and a clear height of 135 ft. for 800 ft. at mid-river.

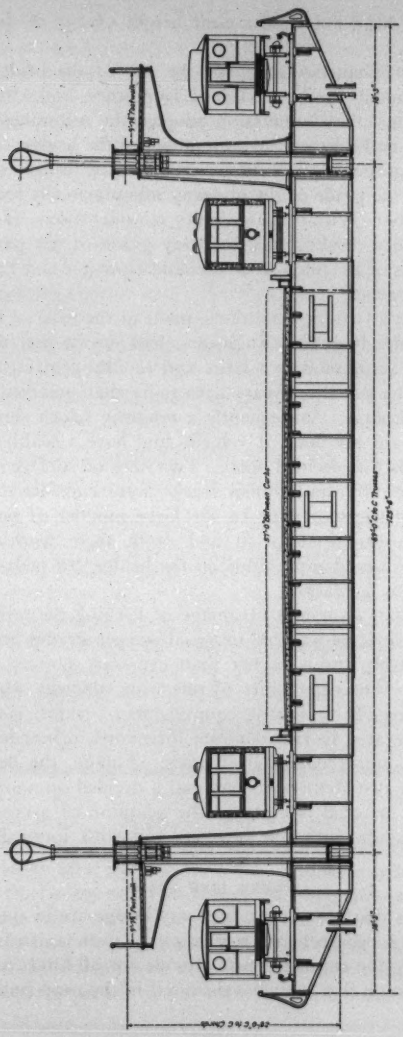
After an exhaustive study of the site of the bridge and its approaches, made by the Board of Engineers, and after several public hearings, the Commission adopted the recommendation of its engineers and selected the present site of the bridge. When the clearance requirements and the terminals of the bridge were determined upon, the grade of the highway was practically fixed within narrow limits. Within these limits considerations of economy and desirability resulted in a roadway grade of 3.5 per cent. on the side spans of the main structure, and from 3.5 to 3.14 per cent. on the approaches.

The study of traffic conditions, made at the time of the report in 1921 by the Board of Engineers, had shown that the bridge will have to accommodate a large and rapidly growing vehicular traffic, and the last three years have more than justified the findings of the Board. Consequently a roadway which should allow ample room for six lines of vehicles and have a width of 57 ft. between curbs was decided upon. Two lines of surface car-tracks and two lines of rapid transit tracks were considered essential for the rapid transportation of the large number of people who will use the bridge daily to and from their working-places. Ample space was also provided on the bridge for pedestrians on business or on recreation.

The obvious economic advantage of keeping the railway lines on as low a level as possible to avoid steeper grades and loss of energy in lifting trains higher than necessary pointed to a one level bridge. The desirability of one wide roadway which could adopt itself readily to hourly requirements of traffic, instead of a split one, operated in two opposite directions, demanded a wide cross-section uninterrupted by trusses. Finally, the definiteness of transverse distribution of load and a decided economy in stiffening truss material determined the adoption of a cross-section with two trusses, with side brackets and with footwalks on an upper level.

ASSUMED LIVE LOADS.

The loads that pass over a highway bridge are in special cases only subject to control and regulation. Given a roadway of a certain width, the engineer must provide for all kinds of vehicles of transportation that may travel over it in the near future. It is



Cross-section of main bridge.

a matter for his judgment to sense the trend in the evolution of vehicular transportation and to meet it in his load provisions. Present-day highway loads are such that horse-drawn vehicles may well be eliminated from consideration. A study of power-driven vehicles manufactured at the present time, of city-street and highway travel, and of state laws regulating motor traffic had led to the conclusion that for a width of roadway of 57 ft. a load of 4000 lbs. per lin. ft. of bridge will represent an excess of 28 per cent. above the present-day average weight of motor travel. Such a load will provide for extraordinary conditions as well as for a possible increase in the weight of vehicles.

The all-steel cars operated on rapid transit lines transporting passengers only in cities and suburbs, with very few exceptions, do not exceed, when fully crowded, 2000 lbs. per lin. ft. of car. The longer cars operated by the Brooklyn-Manhattan Transit in the New York subways weigh about 1900 lbs.; the elevated cars of the Philadelphia Rapid Transit Company weigh 1840 lbs. The electric cars operated by the Pennsylvania Railroad weigh, with seats all filled, 2000 lbs. and, with standees, 2150 lbs. A load of unlimited length of 2000 lbs. per foot of track was adopted as more than covering rapid transit requirements.

The surface cars operated by the P. R. T. weigh about 1230 lbs., the Hog Island cars 1350 lbs. per lin. ft. By adopting 1500 lbs. per foot of track ample provision has been made for a possible increase in the weight of surface cars.

The total live load per lineal foot that possibly could be placed on the bridge is as follows:

Two lines of rapid transit trains at 2000 lbs. per lin. ft. each	4,000 lbs.
Two lines of surface cars at 1500 lbs. per lin. ft. each	3,000 lbs.
57 ft. of roadway at 70 lbs. per sq. ft.	4,000 lbs.
20 ft. of walkway at 50 lbs. per sq. ft.	1,000 lbs.
Total congested load	12,000 lbs.

Such a load of 12,000 lbs. per lin. ft. would represent the bridge loaded to a possible maximum density and all traffic practically at a standstill, and it is fittingly denoted as congested load. Such a density would defeat the object of the bridge, which is to provide room for transportation. To be able to keep moving at a good speed only one-third of this density should prevail. Trains and surface cars require for regular operation with a

much reduced velocity a clear space of at least twice their own length. Because of the possible lengths of station platforms, rapid transit trains have been limited to 500 ft. in strap-hanging New York. A thorough consideration of all factors concerned thus shows that a working load of one-half of the congested load would represent heavily loaded conditions which could occur occasionally only.

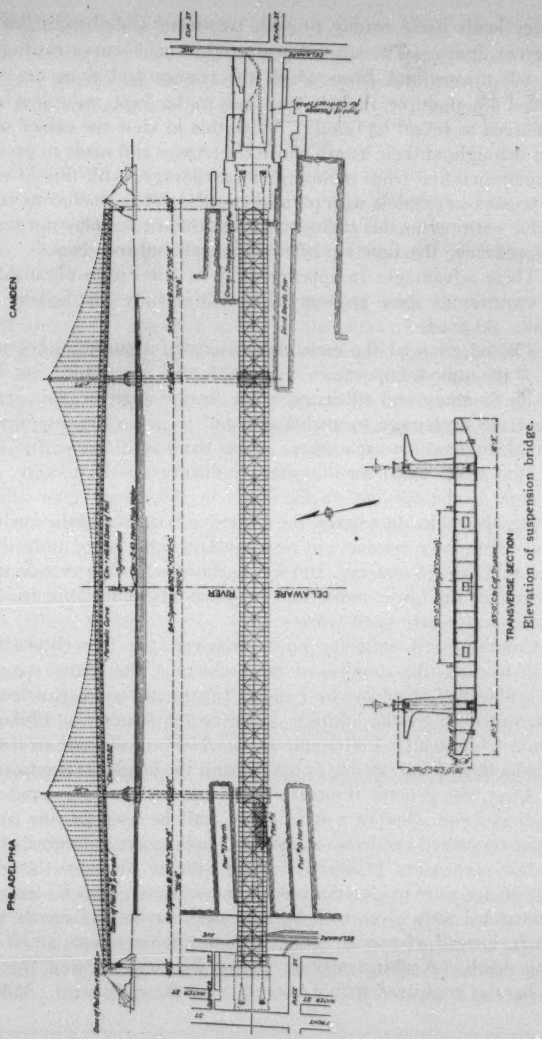
The congested load of 12,000 lbs. per lin. ft. was used in the design of the bridge with stresses correspondingly higher than is usual for bridges. It is well to bear in mind the full meaning of the congested load when making mental comparisons of the stresses in the Delaware River Bridge with those in other bridges. The working load of 6000 lbs. per lin. ft., with stresses corresponding to it, offers a more correct basis of comparison. It is especially important to remember this when the stresses in the stiffening trusses are considered, because the later are subject principally to live load stresses only.

SELECTION OF TYPE AND APPEARANCE.

Based on the required span and the desired capacity of the bridge, the two types of bridges that were available with the foundation conditions existing in Philadelphia and Camden were the suspension and the cantilever. Consequently a complete general design with typical details was made of each type. Both economy and appearance were in favor of the suspension bridge, and after careful consideration the Board of Engineers selected this type.

The Delaware River Bridge is the longest span highway bridge as yet built. It is located in the heart of a great and wealthy city, the cradle of liberty and literature of the country. It spans a majestic river and will tower high above the shore and river traffic. It will be an imposing object for the eye and it should testify to the opulence and broad vision of the commonwealths which built it. It will present the grand gesture of youthful America.

The appearance of the bridge was therefore one of the important considerations in its design. A structure of this magnitude looks best when its beauty of line is chaste and significant of its functions, when its primary organs are shown clear and distinct in their stability and strength. Fortunately, a suspension



Elevation of suspension bridge.

bridge lends itself readily to such treatment and therein lies its inherent fitness. The natural grace of the cable curve resting on the tall towers and from which the trusses and floor are suspended by multiple threads had but to be kept unbroken and unmarred to reveal its beauty. With this in view the cables were kept throughout their length above the trusses and made to present an unbroken line from anchorage to anchorage; with this in view the trusses were made with parallel chords, and as shallow as compatible with reasonable stiffness, so that the trusses may not usurp in appearance the function of main supporting members.

These advantages in appearance, it is true, were obtained at the expense of some economy, but this is fully justified by the results obtained.

The selection of the main dimensions of a suspension bridge are of the utmost importance not only for its appearance, but also for its economy and efficiency. The bridge is an organic structure from anchorage to anchorage, and its main organs exert a mutual influence on each other. Thus the length and stiffness of the side spans exert an important influence on the towers; the amount of the versine of the cable in the centre span affects directly the cable, the towers, the stiffening trusses and the anchorages. A greater versine will result in less cable wire, more truss steel, more tower material, and less anchorage; a shorter side span will reduce the tower motion and require less material in towers, in side and centre span trusses.

Conditions of actuality limit, however, the free choice and direct towards the solution of the problem. The centre span of the bridge was fixed by the Federal permit, the side span length was controlled by the location of Delaware Avenue in Philadelphia. A balancing of economic and æsthetic considerations determined a theoretical versine of 200 ft. and the height of the towers.

After the general dimensions of the bridge were fixed the question arose whether a wire cable shall be used or one made of pin-connected eye-bars or plates. Three general designs of an eye-bar suspension bridge following similar lines as the wire cable bridge were made. Such economic advantages as its features commanded were given to it by increased versines of 220 ft. and 240 ft. instead of 200 ft. The stiffening trusses were all of the same depth. A unit stress of 40,000 lbs. was allowed for the eye-bars as compared with 72,000 lbs. for the cable wire. While,

as expected, the centre span of the eye-bar design was substantially stiffer, the side spans, contrary to expectations, showed slightly greater deflections than the wire cable bridge. In all cases the estimated cost was in favor of the wire cable design.

THE TOWERS.

The towers of a suspension bridge support the cables, the structure hung from them, the live load that may come on the bridge and the wind-pressure against it. The cables rest on properly shaped saddles to which their load and friction holds them immovably.

Under various conditions of loading and temperature the balanced pull in the cable in one or the other of the spans changes and the equilibrium of the system is reëstablished by a change in length of the centre and side spans, in other words, by a horizontal displacement of the points of reaction of the cable. Formerly, when spans were short and loads were light, the displacement was accomplished by placing the saddles, on which the cables rest, on cylindrical rollers, so that they could move on the top of the tower. Experience on large bridges has shown, however, that saddles intended to move do not move before overcoming considerable frictional resistance. This is easily illustrated. The maximum vertical reaction of one cable on its saddle is 24,600,000 lbs under dead load alone. Taking the frictional resistance of the rollers as low as 1½ per cent., it would require a force of 369,000 lbs. to make the saddle move from its dead load position. Much less than this force will bend the tower sufficiently to provide the required displacements. The plain meaning of this is that the tower, even if provided with saddles on rollers, will be subjected to bending as much as with fixed saddles. Besides the element of friction is introduced into the otherwise perfectly elastic system constituting the bridge. This disadvantage has been overcome in more recent years by attaching the saddle positively and immovably to the tower and letting the tower bend. Fixed saddles on bending towers were first applied to the Manhattan Bridge in New York and have since been recognized as the correct treatment for large bridges.

As stated in the above, any change of load on the bridge will result in a displacement of the saddles in the direction of the axis of the bridge. The tower must follow these displacements.

It can do so by bending or by rotating around a pin or a cylindrical rocker-bearing nest located at its lower end. But pins under heavy bearing pressures also offer frictional resistances and are not suitable for movements, and large rocker nests, while reducing these resistances considerably, are quite expensive. To this should be added the cost of bracing to hold the tower in position during erection. Fixed towers subject to bending can be designed to cost not more than rotating towers with the important advantage that the amount otherwise expended on movable parts to allow for saddle displacements is here placed in the metal body of the tower legs. The towers of the Delaware River Bridge were therefore built as fixed towers and the stresses caused in them by bending were provided for in their sectional areas.

The towers were proportioned for a reaction of the cables produced by a dead load of 26,000 lbs. per lin. ft. of main span and 27,000 lbs. of side spans, the congested live load of 12,000 lbs. per lin. ft. and a temperature range of 55° F. above or below closing temperature. A longitudinal wind load of 50 lbs. per sq. ft. was assumed on the transverse projection of the tower and bracing and a transverse wind load of twice the longitudinal projection of the tower. The transverse reaction of the trusses due to the wind on them was determined at 1,200,000 lbs. and that of the two cables at the saddle at 500,000 lbs.

The towers consist each of two columns rigidly braced together to act as a unit. Considering that a tower sustains on top a reaction of nearly 35,000 tons, its transverse strength and rigidity are of great importance. It is desirable that the transverse frame be subjected to very limited deformations. The possibility of an airship striking the tower top at a high speed, while remote, was taken into consideration. It was with this in view that the tower bracing was designed for an additional lateral force of 1,000,000 lbs. applied at the top of the tower and that the bracing members were proportioned for low unit stresses, one-half of those commonly allowed for wind bracing. A rigid frame has thus been attained.

THE DISPLACEMENT OF THE TOWER TOP.

The displacement of the tower top is entirely determined by the balancing of the horizontal pull in the cable in the main and side spans. It is a result of the equilibration of three equilibrium

polygons by the way of elastic deformations. It thus is a function of the moduli of elasticity of the materials concerned. Taking the safe side in either case, the modulus of elasticity of the cable was taken as $E_c = 27{,}000{,}000$ lbs. and that of the tower as $E_t = 29{,}500{,}000$.

The horizontal displacement is computed as follows:

The horizontal pull in the cable for a given condition of load and temperature is computed by the deflection theory formula for this case. This gives at the same time the vertical deflection in the main and side spans resulting from this condition.

From the horizontal pull in the cable the stresses in the cable are determined. The elongations due to these stresses as well as temperature changes furnish the change in length of side span cable. This is equivalent to a virtual change in versine for the same span. The difference in the virtual versine and the one found for the side span balancing condition is taken up by the horizontal displacement of the tower. This is computed by the difference in length of curves and its equivalent horizontal displacement.

This method of computation has been checked independently by the method of successive approximations.

Of the several cases of loading and temperature investigated, full computations were made for four cases. Two of these, Case *B* and Case *C*, were found to produce displacement and vertical reactions which control the combined stresses and the sections in the towers.

The four cases are:

Case A.—Normal and excess fixed load; and congested moving load covering entire bridge at $-55°$ F.

Case B.—Normal and excess fixed load; and congested moving load covering the main span and far side span at $+55°$ F.

Case C.—Normal and excess fixed load; and congested moving load covering the near side span only at $-55°$ F., the main span and far side span being free of moving load.

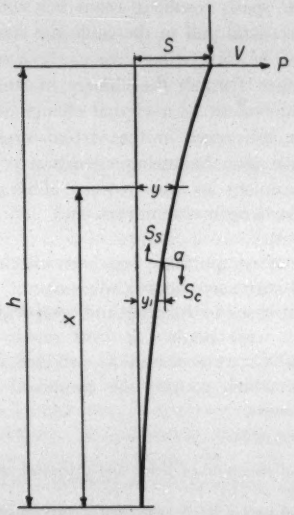
Case D.—Sub-normal fixed load; and congested moving load covering the near side span only at $-55°$ F. By "sub-normal fixed load" is denoted a fixed load of the bridge less than the one computed. Consideration is here given to the possibility of having at some time a lesser fixed load due to lighter flooring or leaving off some of the fixtures which are counted in as weight on the bridge. This sub-normal or least weight has been taken at an extreme low value of 20,000 lbs.

To obtain economy of metal in the required section, the equilization of stresses for the controlling cases is desirable. This has been attained by setting back the top of the tower toward the land 5 inches under normal fixed load and normal temperature.

The total range of horizontal displacement of tower top is for working load 25 in., for congested load 39 in., and for subnormal, and excess and congested load, 44 in.

The determination of the stresses in the towers for any con-

FIG. 1.



dition of loading and resulting horizontal displacement is an ordinary matter of analytical mechanics.

The tower presents the case of a column fixed at one end and compelled to deflect a given distance at right angles to its axis while sustaining a load in direction of the axis. The column is consequently bent by the load and by the compelling force, the two together producing the displacement. Evidently the larger the load, the less the compelling force.

Let V denote the load reaction, P the compelling force, δ the

displacement, the moment at any point above the truss bearings is given by

$$Mx = V(\delta - y) + P(h - x).$$

$$y = \delta + \frac{P}{V}(h - x) - A \sin. nx - B \cos. nx.$$

$$n = \sqrt{\frac{V}{EI}}.$$

A and B are integration constants of the curve of flexure.

At any point below the truss bearings the effect of the bending moment, caused by the truss reactions, must be included. We then obtain:

$$Mx^1 = V(\delta - y) + P(h - x) + (S_c - S_s)(y - y_1) + (S_c + S_s)\,a.$$

$$y = \frac{V\delta + (S_c - S_s)\,y_1}{V + (S_c - S_s)} + \frac{(S_c + S_s)\,a}{V + (S_c - S_s)} + \frac{P(h - x)}{V + (S_c - S_s)} - A \sin. nx - B \cos. nx.$$

As the tower has a variable moment of inertia, it has been divided into sixteen equal divisions and a moment of inertia for the median section of each division was computed. The effect of splices on the stiffness of the tower was taken as equal to an increase of 5 per cent. in moment of inertia of the tower sections. The process of computation begins with the lowest section and finds its top deflection and slope in terms of P. With these as a basis, the deflection and slope of the next section above is computed, and so on. The summation of all the deflections obtained is then equated to the given displacement δ and the equation is solved for P.

With the determination of P for a given displacement, the equation for M_x gives the moment at any point x of the tower.

The analytic method is checked by a method of successive approximations wherein the curve of flexure is at first estimated and then corrected till it agrees close enough with the computed ordinates.

The axial compressive stress due to the vertical load and the bending stress are added algebraically and give the combined stress. The cross-sections of the tower are then proportioned for the combined stresses.

The tower bracing is subjected to the transverse load of 1,000,000 lbs. and the wind load of 500,000 lbs. on top and the direct transverse wind load on the tower columns. The primary as well as secondary stresses for these forces have been computed.

The tower forming a unit, the bracing must participate in any load carried by it. These participation stresses were computed and added to the stresses in the bracing.

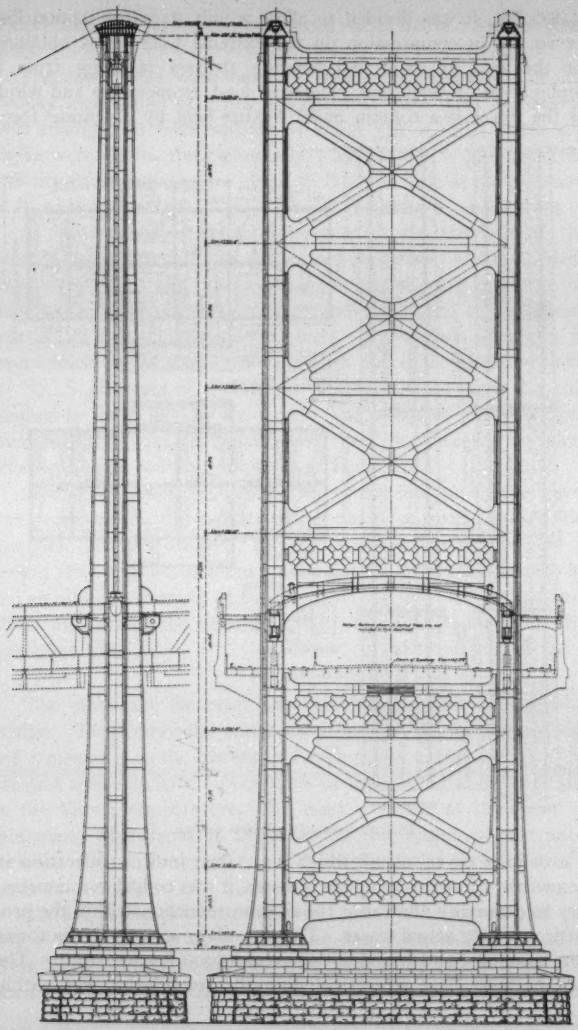
Most of the participation stresses were, however, eliminated during erection by proper cambering of the sections.

MATERIAL AND UNIT STRESSES.

The cross-section of the tower is naturally determined by the cross-section of the bridge, since the clearances for the tracks and the width of the roadway should preferably be maintained throughout the entire length of the bridge. This resulted in a very simple arrangement of two tower legs braced transversely. It was desirable to limit the transverse width of the tower legs to save in the width of the entire bridge and that of the anchorages. Longitudinally the profile of the tower was determined by considerations of strength and looks. From the point of view of strength and economy it was desirable to have a slender column. The bending stresses caused by the displacements of the tower tops are proportional to the width of the leg in the direction of the bending. The narrower the width, the less the bending stress. On the other hand, the radius of gyration of the tower acting as a column had to be kept within reasonable limits. Again, from the point of view of appearance, the tower should present a wide base producing the effect of broad stability. The adopted profile balances well the static and æsthetic requirements. The profile line is formed by hyperbolic curves.

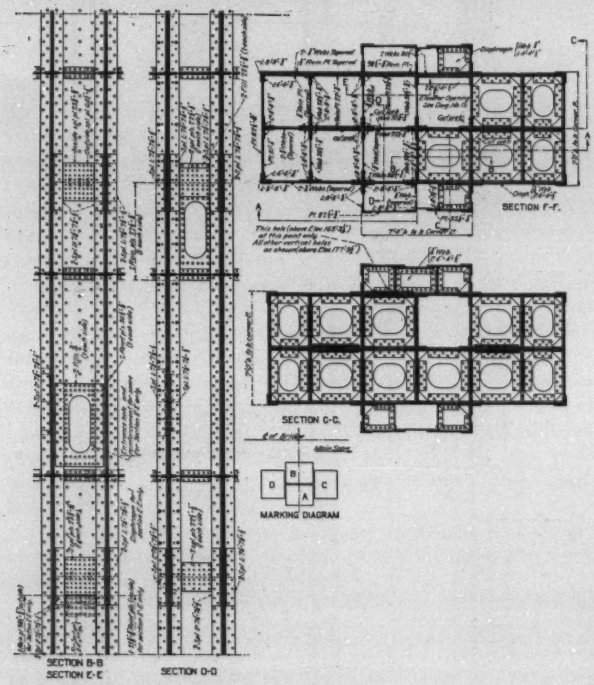
Here again the good appearance of the bridge was obtained at the expense of some economy, and here again the results obtained fully justify the profile chosen.

It has been seen that for a given width of tower certain bending stresses will naturally result, independent of the thickness of the material. It was therefore evident from the start that the use of a steel of higher elastic limit than that obtained in the usual carbon steel is desirable for the tower legs. An excellent material available for this purpose was found to be the structural steel known as silicon steel. It was called for in the specifications to meet the following requirements: Minimum yield point, 45,000 lbs. per sq. in.; ultimate strength, 80,000 to 95,000 lbs.; carbon, 0.4; manganese, 1.0; silicon, not more than 0.45 and not less than 0.20. For this material having a minimum yield point of



Main tower.

45,000 lbs., it was decided to allow a unit stress of 24,000 lbs. per sq. in. in compression on the extreme fibre stress obtained for the sum of axial and bending stresses resulting from a combination of dead load, congested load, temperature and wind. As the tower is a column under flexure bent by a definite force



Typical cross-section of tower column.

to a definite curve, all of which was taken into consideration in the method of determining the stresses, it was considered unnecessary to make any allowance for column reduction within the proportions of the actual tower. The secondary stresses in the tower were computed and found not to exceed 5000 lbs. per sq. in. The total extreme fibre stress including the secondaries was within

27,000 lbs. The cross-sectional area varies from 1800 to 3000 sq. in.

The tower legs were built of cellular construction. A leg consists essentially of three longitudinal ribs with transverse webs arranged to form squares of about 3½ ft. All plates and angles were ¾ in. thick generally. The tower is 343 ft. from masonry to base of saddle. It is 38 ft. 8 in. wide at the base and 12 ft. wide at the top.

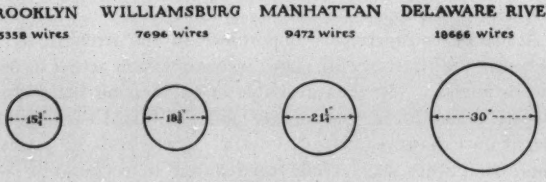
A matter considered of importance in the strength of the tower was the resistance of plates in compression acting as webs between angles. The data available in engineering literature of tests made heretofore was meagre and the Board of Engineers ordered an exhaustive series of tests. These tests of plates in compression were successfully carried out in coöperation with the U. S. Bureau of Standards. They proved that the rule adopted in the design of the tower: The width of plates in compression not to exceed thirty times their thickness in the clear between angles, was conservative and justified.

As already stated before, the transverse bracing of the tower was proportioned for rigidity rather than for strength. As the computed stresses are low, the material used in the bracing is carbon steel with a minimum yield point of 37,000 lbs. per sq. in. and an ultimate strength of 62,000 to 70,000 lbs. All diaphragms, fillers, etc., are of the same material. The total tonnage of steel was 9800 tons.

THE CABLES.

The cables are the most important members of a suspension bridge. They carry the entire dead and live load of the bridge and transmit it to the towers and anchorages. The cable gets its greatest stress under a full continuous load over the entire bridge at the lowest temperature. Its least stress is at its crown at mid-span, wherefrom it increases as the secant of the cable inclination towards the tower. The adopted cross-section logically leads to two cables, two tower legs and two trusses, resulting in a substantial economy in material and cost. Heretofore the larger suspension bridges sustaining heavy loads, like the three New York suspension bridges, were built with four cables. The latest and heaviest of these, the Manhattan Bridge, has four cables

of 20½-in. diameter each. In the design of the Delaware River Bridge two cables of 30-in. diameter are used. This daring advance of practically doubling the area of one cable beyond the heaviest laid was the idea of the Chief Engineer, Mr. Modjeski. His judgment has been fully justified in the results of the cable as actually laid. No extraordinary difficulties were encountered either in the manufacture or the erection of the cables.



Cable sections of various bridges.

For a dead load of 26,000 lbs. per lin. ft. of centre span and 27,000 lbs. per lin. ft. of side spans and the congested live load of 12,000 lbs. per ft. of bridge at 55° F. below closing temperature, the horizontal component of the pull in one cable is:

Due to dead load	$H = 24{,}700{,}000$ lbs.
Due to live load and temperature	$H = 10{,}900{,}000$ lbs.
Total	$H = 35{,}600{,}000$ lbs.

The greatest secant of the cable is 1.127 and the greatest stress in one cable is 40,110,000 lbs.

The cable consists of 61 strands of 306 wires each. The total number of wires is 18,666 wires. Each wire galvanized has a diameter = 0.196 in., and the total area of a cable is 562 sq. in. The greatest unit stress, therefore, becomes 71,500 per sq. in. of galvanized wire.

For a continuous and full working load of 6000 lbs. per lin. ft. of bridge the same conditions will produce a greatest pull in one cable of 34,180,000 lbs. and a unit stress of 60,800 lb. per sq. in.

The specifications for the wire of the cable required a minimum yield point of 144,000 lbs. per sq. in. and an ultimate strength of 215,000 lbs. per sq. in. The wire actually furnished by the American Cable Company, sub-contractors to the Keystone Construction Company, showed an average ultimate strength of 223,000 lbs. per sq. in.

The cable rests on the tower on cast-steel saddles. The saddles have a base length of 13′ – 4″, and the bearing surface is curved to a radius of 16 ft. The surface is grooved to fit the shape of the individual strands. The bearing pressure exerted by the wire on the saddle has been computed under a wide range of assumptions to lie within 1600 to 2800 lbs. per lin. in. of wire. Tests made on the same diameter wire applying squeezing pressures of 1000 to 4300 lbs. per sq. in. of wire; while at the same time, pulling the wire to a breaking load showed no change in the strength of the wire due to squeezing pressure.

The range of bending of the cables near the towers under the greatest deflections of the trusses and towers will not exceed one degree either way.

The trusses and floor are hung from the cables by means of wire rope suspenders at every panel point. Cylindrical steel castings are tightly drawn together so as to exert sufficient friction to prevent their sliding. Around these castings the suspenders rest. It is therefore of the greatest importance to consolidate the cable to as dense a cylinder as possible and to so bore the castings that they may fit tightly when bolted together.

From previous experience with the Williamsburg and Manhattan Bridge cables, and a test cable made by the Roebling Company, the diameter of the cable was computed to be 29⅞ in. The cable contract called for a test cable of full section and 5 ft. long to determine the exact diameter to which the actual cable shall be compacted at the point of attachment of the castings. The compacting test made on this cable piece resulted in the diameter of the cable to be 29⅞ in. as prefigured. The compacting was done by a squeezing machine containing six hydraulic jacks of 30 tons capacity each, exerting a pressure on the cable of 1000 lbs. per sq. in.

At each panel point there are two suspender ropes looped around the cable casting, making thus four wire ropes per panel point. The ropes have a diameter of 2¼ in. and are specified to have a strength of 400,000 lbs. direct stress and 380,000 lbs. over a sheave of $d = 32$ in. The greatest pull of the suspenders at the panel point will not exceed 400,000 lbs. for the four ropes. A surplus of strength was given to the suspenders so as to enable them to sustain heavier loads should it be desirable in the far future to increase the traffic capacity of the bridge.

The tonnage of steel in the cables, suspenders, and anchorages, is:

Wire in cable	6,800 tons
Anchorage steel	2,850 tons
Steel castings	500 tons
Wire rope	450 tons
Total	10,600 tons

THE ANCHORAGE CABLE BENTS.

In accordance with the architectural outline mentioned in the above, the cables extend above the trusses and continue for some 35 ft. beyond the end of the side spans and then bend over an inclined steel tower descending to the anchorage bars at an angle of about 45 degrees. The cable rests on a saddle similar to the main tower saddles but smaller. The greatest reaction of the cable on the saddle is here 23,200,000 lbs. or two-thirds of that on the towers. The inclined steel tower is aptly called the anchorage cable bent and is inclined 27 degrees from the vertical. It is 76 ft. long, 8 ft. wide and 7 ft. deep. Its function is to transmit the cable reaction to the anchorage masonry. Both during erection and afterwards the portion of the cable and anchorage bars located beyond the bent stretch with the advent of a load on the bridge. The stretch under dead load and temperature is 1.9 in. and under greatest live load 0.5 in., or a total of 2.4 in. Here again the saddles must undergo a displacement to allow for the stretch. This can be accomplished by placing the saddle on roller or rocker bearings or attaching them immovably to the bent and allowing the latter to bend. The same reasoning as was applied to the towers applies here, and the tower bents were therefore designed to deflect with the pull of the cables.

Additional considerations, however, prevail here. By taking down the cable reaction by a long steel post, considerable masonry is saved and the pressure on the toe of the anchorage is thereby reduced. Of still greater importance is the fact that the cable seat is at a height of about 25 ft. above the track level and not more than the width of 7 ft. is available for a masonry support of the saddles, the same as the width of the tower legs. It is held that such a comparatively narrow masonry support does not offer the same security against an accidental lateral force as is provided by a steel post of the same width. The bents were made of silicon steel. The cross-sectional area is 1283 sq. in. per post.

The bent was designed for a lateral force of 250,000 lbs. on each saddle and a lateral wind reaction from the truss of 320,000 lbs.

THE FLOOR SYSTEM.

The floor system of a bridge provides support for the tracks of railways and for the pavement of highways. Here it has been made of simple design. To distribute the load over the cable uniformly and to approach as much as possible a continuous curve in the cable, the suspender points should be as near as reasonable to each other. Short panel lengths are, therefore, desirable in wire cable suspension bridges. In the Delaware River Bridge a panel length of 20.5 in. was found to be suitable.

For this short panel length a floor system has been designed consisting of a floor beam 8 ft. deep between trusses and of brackets outside of the trusses. Between the floor beams and riveted to them extend stringers. The roadway stringers are 18-in. I-beams placed 3 ft. 10 in. centres. On top of these I-beams a reinforced concrete slab will be laid covering the entire width of the roadway, 57 ft., between curbs. An asphalt paving will be laid on the concrete slab.

The reinforced concrete slab will be 6 in. thick and presents a point of considerable interest. The heavy motor truck loads of to-day and the probable increase which may be expected in the future, are provided by assuming 30-ton trucks on the roadway. This calls for 15,000 lbs. on a wheel to which a 50 per cent. dynamic increment is added, making a total of 22,500 lbs. Such heavy concentrations require considerable thickness of concrete floor. But thick floors are not desirable in long-span bridges because of their weight. To obtain sufficient strength in a thin concrete slab it is necessary to provide definitely for steel reinforcement in shear. Also it is further necessary to have positive assurance of the accurate placing of the top and bottom reinforcing rods. With this in view, the concrete slab will be reinforced by an integral truss of reinforcing steel 4½ in. out to out of steel, leaving a cover ¾ in. of concrete top and bottom. The truss members will be positively connected to the chords. The reinforcing trusses will be placed 6 in. centres and transverse to the bridge. So designed the concrete slab will furnish a comparatively light and amply strong continuous foundation for the roadway pavement.

THE STIFFENING TRUSSES.

The function of the stiffening trusses, as is implied by their name, is to stiffen the otherwise flexible cable. They are intended to distribute more or less uniformly the discontinuous and concentrated live loads that come onto the bridge. The total dead load of the floor system and that of the trusses themselves is supported by the cables through the suspenders. So that the cables, the towers and the stiffening trusses form an elastic system in equilibrium under their own load and closing temperature. At that time the stiffening trusses are not stressed at all. With the advent of live load and change of temperature, the elongation of the cable results in the deflection of the bridge and in a newly formed elastic system in equilibrium. The distortion of the original system is resisted by the weight of the entire structure and the rigidity of the trusses. It is the part of good design to select trusses to be sufficiently rigid for the character of travel without consuming too much material. Great depths of trusses add only a small proportion to the stiffness of a suspension bridge. With these considerations in view and with due allowances for clearances, the depth of 28 ft. was adopted for the stiffening trusses.

The members in stiffening trusses are subject to reversal of stresses. The Warren type of truss, or the simple triangular system, has been selected because it is so well adapted to these reversals.

THE EVOLUTION OF STIFFENING TRUSS THEORY.

The suspension bridge is a system in which vertical forces cause horizontal pulls at its points of support. But no such considerations and definitions were in the mind of the savage and barbarian when he was compelled to bridge a river. His reasoning was inductive; he learned from his surroundings. The spider thread was example enough to stretch a rope between two points. Light suspension bridges made of ropes or iron chains have been known for many centuries. Kircherus in his "China Illustrata" mentions in 1667 a bridge consisting of twenty chains. Light suspension bridges began their real development in the beginning of the last century, when Jacob Finlay built a number of such bridges. In the year 1816 Telford built the Menai Bridge with a span of 580 ft. The bridge over the Danube at Budapesth had a span of 666 ft. and the Clifton Bridge near Bristol, England, one of 702 ft. These were all iron-chain bridges.

Led by the consideration that iron drawn in wire form develops higher strength, Messrs. White and Hazzard in 1816 "constructed a foot bridge across the river at the Falls of Schuylkill, above Philadelphia, whose cables were made of six wires, ⅜ in. diameter, and so a new epoch of development was instituted. The span of this structure was 408 ft.; its cost was $125; a toll of one cent was charged for passage, and only eight passengers were allowed upon it at one time."

In 1826 the Séguin Brothers in France made a series of careful experiments on wire for cables. Their work was approved in a report to the Paris Academy signed by Prony, Frésnel, Mollard, Girard. A number of wire cable bridges were then built in European countries. Notable among them are the Freibourg Bridge in Switzerland, built in 1832, with a span of 895 ft., and that at Roche-Bernard, France, with a span of 633 ft. All these bridges had as a common defect a lack of rigidity and oscillations produced by wind. Such stiffening as was incidentally provided in them consisted of bridge railings. The lack of rigidity led to the introduction of the stiffening truss.

The stresses in the flexible suspension bridges were well known to the able engineers who planned and built them. The great Navier wrote a report on these bridges in 1823. The effect of the dead load of the bridge was evidently known to the older engineers. In fact, the only stiffening provided was its own dead load. The deflection under live load was too noticeable to be neglected and was, in fact, in later years made part of the analysis. With the advent of the suspended stiffening trusses Rankin in England and Ritter in Switzerland, at about the middle of the last century, developed the theory of the rigid stiffening truss. According to the assumptions of this theory, the suspended truss remains absolutely rigid under the live load, and the original curve remains undisturbed. This assumption by itself tacitly eliminates the effect of the dead load of the bridge. As spans were comparatively low and trusses were easily made very stiff, the error so committed, while substantial, was not serious. In later years, with the growth of the knowledge of elastic structures, the stiffening truss began to be treated as an elastic member. Mueller-Breslau and Melan developed to a high degree the elastic theory of suspension bridges. Perhaps, because few and comparatively small suspension bridges were built in Europe at that

time, the effect of deformation in these bridges was not given due attention. At any rate, the original assumption contained in the Rankin theory, that the cable curve remains undisturbed, was tacitly taken over by the elastic theory. In the eighties of the last century Melan in his work on suspension bridges pointed the way towards a more exact theory, the importance of which he himself, however, did not fully realize. In fact, the speaker received from him the same impression in a personal interview at Prague in 1908. At the beginning of this century, while working on the design of the Manhattan Bridge, the speaker first realized the economic and engineering importance of this theory for long-span suspension bridges. He then developed a complete set of formulas covering all required cases, known under the more fitting name of the deflection theory, and applied it for the first time to the design of the Manhattan Bridge.

THE DEFLECTION THEORY.

The great importance of the deflection theory is not a mathematical one. While resulting in greater accuracy, its chief merit in comparison with the elastic theory is not that of greater accuracy but that of very substantial economy. In fact, the elastic theory of suspension bridges is so far in error that its use for large bridges is out of the question. It is not even approximately correct.

The main considerations of the deflection theory can, I believe, be easily explained. The bridge is considered as a balanced elastic structure under dead load. When it deflects under live load or temperature the distortion of the cable is taken into account. When a live load is placed on the bridge its weight is transmitted to the cables, which elongate, due to the increased stress in them. As a result the towers bend forward and the trusses deflect, resulting in a new system of equilibrium. In this new system the cable is distorted from its original curve. The analysis of stresses in the various parts is made on the basis of the new form. Since the distortions of long span bridges are appreciable, their effect on the stresses in the structure is important. In so much the deflection theory provides a greater degree of accuracy. But the important feature of the theory is the fact that while considering the distortion of the cable curve, it also necessarily has to consider the effect of the dead load of the

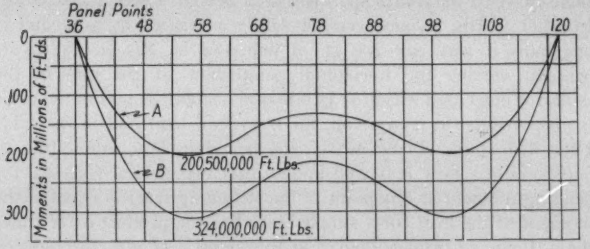
bridge in order to obtain the new equilibrium. This can be illustrated by an example.

Let a live load be placed on the centre span sufficient to cause a downward deflection at mid-span of 5 ft. The newly distorted cable curve of the centre span will then be 5 ft. lower at its crown. In other words, its new versine will be increased by 5 ft. and if originally it was, say 200 ft., it will now be 205 ft. With its original versine the horizontal component of the pull in the cable for dead load was $H = 25{,}000{,}000$. Since the pull in the cable is proportional to its versine, the new dead load pull in the cable after distortion will be nearly 2½ per cent. less, amounting to 610,000 lbs. Now it is well known that the general formula for bending moment at any point in the stiffening truss is equal to the moment of the load for a simple beam less the product of H times the ordinate of the cable curve at that point. If now the reduced dead load H of 610,000 lbs. will be multiplied by the new ordinate of the cable curve, which at the centre is 205 ft., a moment of 125,000,000 ft. lbs. will be obtained. This moment acts as a relieving moment in as much as it reduces the bending moment in the truss. The same is true of the live load H which contributes its share to the stability of the polygon and proportionally reduces the moment in the truss. The large amount of the relieving moment illustrates sufficiently the economic importance of this theory of computation.

It is now seen that the heavier the bridge, the greater is the effect of its dead load in reducing the bending moments in the trusses. This brings us back to the common experience that the heavier a suspended chain is, the more difficult it is to distort. The merit of the deflection theory consists thus mainly in resurrecting the knowledge of the effect of the dead load on the structure which was known to the early builders of suspension bridges and was neglected and forgotten by the theories of Rankin, Ritter, and the elastic theory.

The mathematical treatment of the theory consists of equating the work of the external forces acting on the cable to its internal work of deformation. The cable curve being assumed parabolic under dead load becomes after distortion, being controlled by the stiffening trusses to which it is tied, a bastard curve. It is a cross between the parabola and the neutral curve of a beam. It belongs to the catenarian family. The computation, when car-

ried out, has the advantage that the deflection of the truss or of the cable at any point can be very readily determined. In fact, the whole theory is based, as we have seen, on the deflections of the system. Therefore, its name.

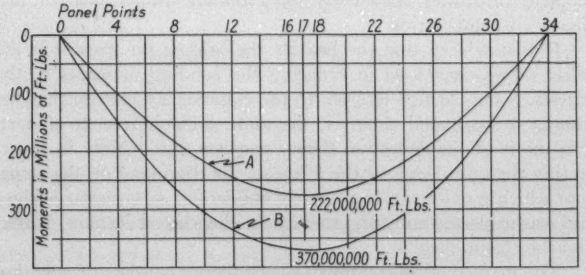


Positive moments in one centre span truss.

COMPARISON OF DEFLECTION AND ELASTIC THEORIES.

The speaker has made a comparison of the deflection theory and the elastic theory as applied to the Delaware River Bridge. The results given for both theories were obtained by the use of identical data.

The four accompanying diagrams show the bending moment

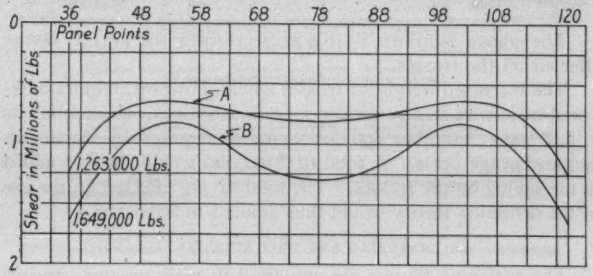


Positive moments in one centre span truss.
Comparison of moments and shears. Curve *A* shows results by deflection theory. Curve *B* shows results by elastic theory.

and shear curves for the centre and side spans plotted for both theories. The areas enclosed within the two curves represent the measure of the theoretical saving effected in the stiffening trusses. For the centre span the application of the elastic theory would

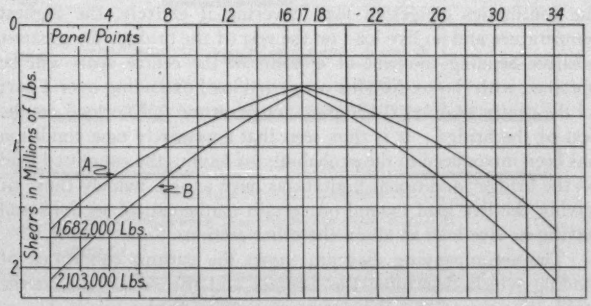
require theoretically 55 per cent. more material for the chords and 43 per cent. for the diagonal web members; for the side spans it would require 40 per cent. for the chords and 26 per cent. for the web members.

In the actual design of the truss members certain minima



Positive shears in one centre span truss.

sections had to be used for practical considerations. These reduce the theoretical saving to the actual saving of 51 per cent. for the



Positive shears in one side span truss.
Comparison of moments and shears. Curve *A* shows results by deflection theory. Curve *B* shows results by elastic theory.

chords of the central span and 43 per cent. for its web members. The actual saving for the chords of the side spans is 38 per cent., and for the diagonal web members 24 per cent.

The total saving in steel is 3211 tons, or 42 per cent., of the total amount used in the trusses proper.

Based on the contract prices obtained for the stiffening trusses, the cost of the 3211 tons is:

Special steel—5,024,000 lbs. @ 8.75c.	$440,000
Silicon steel—1,244,000 lbs. @ 7.85c.	98,000
Carbon steel— 154,000 lbs. @ 6.90c.	10,000
Total	$548,000

The above sum of $548,000 represents the direct saving effected on the trusses.

The increased weight of trusses would, however, require additional cable wire and suspender ropes to sustain it, which amounts to $206,000. Another $96,000 would be required by the towers and anchorage bents. A total of $302,000 would then be added to the saving on the trusses. The total saving effected by the use of the deflection theory would thus amount to $850,000.

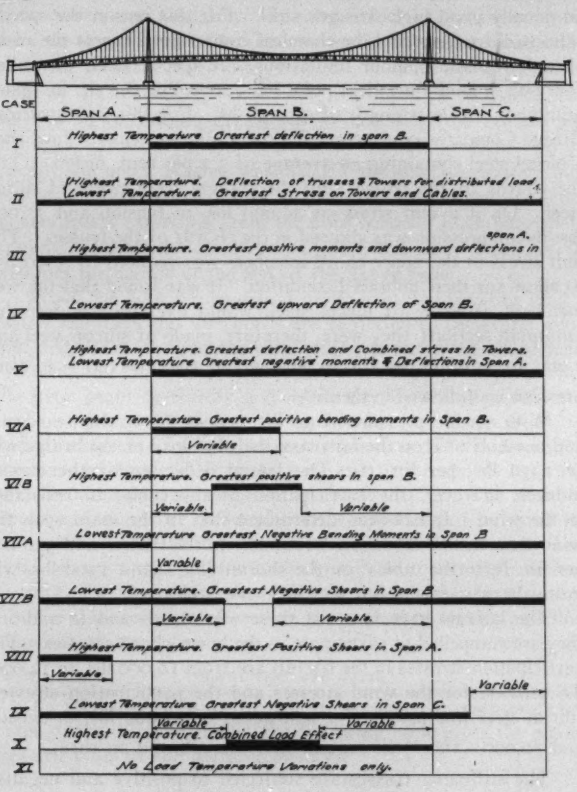
MATERIAL AND UNIT STRESSES.

The stiffening trusses are subjected to their greatest stresses under a combination of live loads acting on the entire structure and distributed in various ways for various points. Thus the greatest positive moments in one side span will be obtained with the continuous congested load covering it entirely, the highest temperature and no live load on the rest of the bridge; the greatest positive bending moment at a point in the centre span will be obtained with the continuous congested load extending over a part of the centre span, at the highest temperature and no load on the rest of the bridge. It is thus seen that an entirely new condition has been introduced to the probability of having the congested load on the bridge; additional limitations have arisen, namely those of having the live load extend on certain mathematical sections and having no live load at all on the other sections.

The accompanying diagram shows the various conditions of loading which determine the stresses and the deflections in the stiffening trusses of a three-span suspension bridge.

While there is a possibility that under extraordinary conditions such loadings which cause the greatest stress in the various members may occur, it is highly improbable that they will occur. The design of the bridge has, however, provided for this contingency by adopting comparatively high but safe unit stresses. Under ordinary conditions the live loads on the bridge will be more or less uniformly distributed along the spans and the duty of

the stiffening trusses will be but a fraction of their provided strength. The use of a high-strength steel is, therefore, here but a logical conclusion. While the trusses are subject to moderate



Suspension bridge; conditions of loading.

stresses under ordinary conditions, they are given a reserve strength which provides for the wide range and high values that might occur.

An excellent steel of high strength was available for this

purpose. It is known as nickel steel and has been successfully used on the Manhattan and other long-span bridges. The opportunity was, however, offered to steel manufacturers to furnish an equally good high-strength steel. For this reason the specifications did not specify the chemical composition except the usual phosphorus and sulphur limitations. A special steel was called for with a minimum yield point of 55,000 lbs. per sq. in. and a minimum tensile strength of 90,000 lbs. For this the American Bridge Company, contractors for the stiffening trusses, furnished a nickel steel containing an average of 3.2 per cent. nickel. Only the chords of the trusses and the rockers were made of nickel steel. On it a unit stress of 40,000 lbs. in tension and 35,000 lbs. in compression was allowed in the chords of the trusses. The unit stress in the upper chord members was reduced by 3000 lbs. to allow for their unbraced condition. It was found that the web members, if made of nickel steel, would overrun the desirable minimum sections, they were, therefore, made of silicon steel and a unit stress of 32,000 lbs. in tension and $32{,}000 - 140\,\frac{1}{r}$ in compression was allowed in them.

With a wind pressure of 30 lbs. per sq. ft. on the near truss and one-half of it on the far truss, the wind load on the bridge will be 1350 lbs. per lin. ft. The lateral deflection of the trusses induces, however, the participation of the cables in resistance to the wind. It has been determined that in the main span the wind load will be distributed 1000 lbs. to the trusses and 350 lbs. per lin. ft. to the cables; on the shorter side spans, 1300 lbs. will go to the trusses and but 50 lbs. to the cables.

The laterals have to resist these wind loads and in addition they are compelled to participate in the lower chord stresses. The participation stresses in the laterals are from 10,000 lbs. to 14,000. To provide for the wind stresses and the participation stresses, silicon steel was used and a unit stress of 32,000 lbs. in tension and $30{,}000 - 100\,\frac{1}{r}$ in compression was allowed on them.

The stiffening trusses are subjected to positive and negative moments which cause reversal of stresses in the truss members. Because these changes take place more or less gradually and because of the improbability of ever attaining their maximum stresses, the members were proportioned for the greatest stress and no additional allowance was made for stress reversal.

THE UNBRACED TOP CHORDS.

The cross-section of the bridge made the spacing of the trusses 89 ft. c. to c. The usual lateral bracing for the top chords proved to be clumsy and unsightly. It would obstruct for the motorist the free view of the bridge and towers and would require bulky members to carry their own weight. A study was, therefore, made of the advisability of omitting the upper chord lateral bracing and of restraining the chords by means of the vertical web members. It was found to be perfectly feasible to do so. Consequently the upper bracing was omitted and the trusses were built so that the top chords are held by the vertical web members acting as cantilevers extending from the floor beams. Each panel point presents a channel-shaped section with the floor beam as the web and the chords at the end of the flange ends.

When the floor beams deflect under load, the verticals tend to deflect sideways and the chords are subjected to transverse bending. At the same time, the compressive force in the chord tends to buckle it. Similarly the brackets when loaded tend to deflect the floor beam outwardly and again the top chord is subjected to bending and buckling. A combination of these cases for various numbers of panels has been studied, and it was found that the average stress that may be produced in the chords by transverse bending does not exceed 3000 lbs. per sq. in. The allowable stress was, therefore, reduced for the upper chord by 3000 lbs. per sq. in. An elastic and safe arrangement has thereby been obtained at a cost substantially lower than that of overhead bracing. Besides, a free view from the roadway has been secured.

The total tonnage of steel in the stiffening trusses and floor system is 18,700 tons.

QUANTITIES AND COSTS.

To furnish a scale with which to measure the importance and value of the discussed items, the following table gives the quantities and costs of the portions included:

	Material.	Cost.
Main Towers:		
Silicon steel	12,941,000 lbs.	$961,500
Carbon steel	6,349,000 lbs.	420,900
Cast steel	322,000 lbs.	58,200
	19,612,000 lbs.	$1,440,600

	Material.	Cost.
Anchorage Cable Bents:		
Silicon steel	1,959,000 lbs.	182,200
Carbon steel	676,000 lbs.	62,900
Cast steel	835,000 lbs.	97,700
	3,470,000 lbs.	$342,800
Cables and Suspenders:		
Wire cables	13,561,000 lbs.	2,441,000
Suspender rope	905,000 lbs.	255,700
Anchorage steel	4,720,000 lbs.	224,200
Steel castings	1,010,000 lbs.	227,000
Miscellaneous steel	198,000 lbs.	18,000
	20,394,000 lbs.	$3,165,900
Stiffening Trusses:		
Special steel	11,300,000 lbs.	988,800
Silicon steel	9,100,000 lbs.	714,300
Carbon steel	16,700,000 lbs.	1,152,300
Miscellaneous steel	170,000 lbs.	32,600
	37,270,000 lbs.	$2,888,000
Total steel	79,948,000 lbs.	Total cost. $7,655,800

The average cost of the steel is 9.85 cents per pound, or roughly, 10 cents.

CONCLUSIONS.

The engineers of the Delaware River Bridge intend to provide a structure for the demands of the present and for the vision of the future. The bridge is designed to accommodate all existing travel and to take care of its future expansion. They endeavored to employ the best engineering science known to them and to utilize in the most suitable manner the materials available for construction, so that the bridge may be a measure of our time and an expression of its spirit.

Such degree of success as has been attained is due to the creative genius and the fine spirit of coöperation of the Board of Engineers and its engineering staff.

THE MEASUREMENT OF ACOUSTIC IMPEDANCE WITH THE AID OF THE TELEPHONE RECEIVER.*

BY

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OBJECT OF THE PAPER.

It is proposed to describe a method of measuring the acoustic impedance [1] of solids or gases in the form of discs, or cylinders, to the passage of plane waves of sound. The method is based upon the reaction of the acoustic conductor upon the vibrations of a telephone receiver diaphragm which is used as the source of plane waves in a tube (Figs. 2 and 8). This reaction in the receiver is measured electrically, by connecting its coils in the fourth arm of a Rayleigh bridge (Fig. 3). The method also depends upon certain analogies between uniform electric-line conductors carrying alternating currents, and acoustic-tube conductors carrying sound.

Only a few measurements of acoustic impedance measured by this method are at present available, and the theory may have to undergo modification, as numerical data are accumulated, but the method itself is definite, and should give interesting results in comparison with other methods of acoustic measurement.

It may first be advantageous to outline the analogies between an electric-line conductor and an acoustic-tube conductor. The experimental facts concerning the electric line are well established. The corresponding experimental facts concerning the acoustic tube are still only imperfectly known, and the acoustic results can therefore only be depended upon, at present, to a first approximation.

SUMMARY OF ESSENTIAL FACTS CONCERNING A UNIFORM ALTERNATING-CURRENT LINE CONDUCTOR.

In Fig. 1, AB represents a uniform single-wire electric line L km. long, having a linear conductor impedance of x ohms $\angle$ per wire km. and a linear dielectric admittance of y mhos $\angle$ per wire

* Address delivered Thursday, September 18, 1924, on the occasion of the celebration of the centenary of the founding of The Franklin Institute.

[1] Bibliography 2 (pp. 245, 262 and 267) and 9.

km. The line is grounded at B through an impedance of σ ohms $\angle$, and is energized at A by an alternating e.m.f. of E_A volts * rms. at a single frequency of f cycles per second. The values of z and y are definite known complex functions of the frequency. The line possesses, at this frequency, a certain characteristic impedance, z_0, ordinarily a few hundred ohms in value, which is briefly called its surge impedance, a complex quantity. Then the impedance offered at the generator end A is known [2] to be

$$Z_A = z_0 \tanh \delta_A \qquad \text{ohms } \angle, \text{ or vector ohms} \qquad (1)$$

where δ_A is the "position angle" at A, measured in hyperbolic radians (complex). This position angle at A is the sum of the angle θ subtended by the line at this frequency of operation and the position angle δ_B at B: Such that

$$\theta = \theta_1 + j\theta_2 = L\sqrt{zy} = L\alpha = L(\alpha_1 + j\alpha_2)$$
$$\text{hyp. radians or hyps. } \angle \qquad (2)$$

$$\tanh \delta_B = \frac{\sigma}{z_0} \qquad \text{numeric } \angle \qquad (3)$$

or $$\delta_B = \tanh^{-1}\left(\frac{\sigma}{z_0}\right) \qquad \text{hyps. } \angle \qquad (4)$$

and $$\delta_A = \theta + \delta_B \qquad \text{hyps. } \angle \qquad (5)$$

The conception of a position angle is convenient, because, in the steady state, at any point along the line, the voltage is directly proportional to the sine, the current strength to the cosine, and the impedance beyond, to the tangent of the local position angle, as has been demonstrated by numerous tests.

If the line is directly grounded at B, formula (3), with $\sigma = 0$, gives $\tanh \delta_B = 0$, and thus $\delta_B = 0$. Consequently, $\delta_A = \theta$ in such a case. The current strength at A is then

$$I_A = \frac{E_A}{z_0 \tanh \theta} \qquad \text{amperes } \angle \qquad (6)$$

If, however, the line is freed or opened at B, formula (3) with $\sigma = \infty$, gives $\tanh \delta_B = \infty$, and thus $\delta_B = \frac{j\pi}{2}$. That is, the position angle at B is an imaginary hyperbolic angle, or simply a circular

* The contraction rms. stands for root-mean-square, and unless otherwise specified, voltages and currents will be assumed throughout as rms. values.

[2] Bibliography 3.

angle of $\frac{\pi}{2}$ radians, or 1 quadrant of arc. The current at A then becomes

$$I_A = \frac{E_A}{z_0 \tanh\left(\theta + \frac{j\pi}{2}\right)} = \frac{E}{z_0 \coth \theta} \quad \text{amperes} \angle \qquad (7)$$

It may be noted that if any hyperbolic angle θ is increased by $\frac{j\pi}{2}$, the tangent of the new angle is always equal to the co-tangent, or tangent-reciprocal, of the original angle θ.

In any intermediate case between grounding and freeing at B, when the impedance σ is inserted, the position angle δ_B at B becomes complex, and δ_A is increased above θ by vector addition of θ and δ_B, as in (5).

The behavior of any such uniform line, under a steady impressed frequency, will be completely determined if we know its two constants, the vector surge impedance z_0 and line angle θ. The easiest way to find these is to measure the impedance offered by the line at A, when freed and grounded at B, respectively. These impedances are, by (6) and (7),

$$R_{af} = z_0 \coth \theta \qquad \text{ohms} \angle \qquad (8)$$

and

$$R_{ag} = z_0 \tanh \theta \qquad \text{ohms} \angle \qquad (9)$$

The product $R_{ag}\, R_{af}$ is then equal to z_0^2, the square of the surge impedance, and the ratio R_{ag}/R_{af} is equal to $\tanh^2\theta$, the square of the line angle's tangent. Tables and charts of complex hyperbolic functions are then available [3] for determining θ.

APPLICATION OF HYPERBOLIC FUNCTIONS TO A UNIFORM ACOUSTIC-TUBE CONDUCTOR.

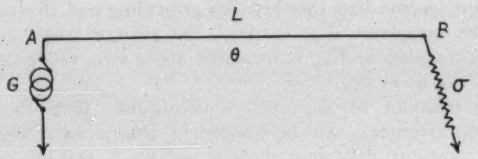
According to measurements made in the laboratory [4] there is a close analogy between the behavior of an acoustic air-tube conductor, under an impressed vibromotive force, and that of an electric-line conductor, under an impressed electromotive force. A uniform fibre tube of smooth internal wall, AB (Fig. 2), has, at the end of A, a telephone receiver T, of which the cap has been opened up to admit the tube. The telephone receiver acts as a source of substantially plane sound waves, that are produced by exciting the coils of T with an alternating current of steady

[3] Bibliography 6.

[4] Bibliography 10, 12, 14 and 15.

known frequency. A smooth disc or plug, P, may be set, friction tight, at any desired point in the tube. Under these conditions, powerful standing waves of sound are set up along the tube line or air-column AB, which resemble the standing electric waves set up along the conducting line AB of Fig. 1. The electric wave propagation velocity in Fig. 2 is often not far from 300,000 km. per second; so that under a telephonic frequency of say 1000 ~,

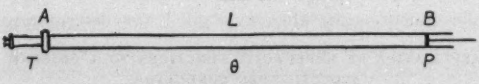
FIG. 1.



Electric line conductor carrying alternating-current waves.

the wave-length λ, obtained by the formula $\lambda = v/f$, is roughly nearly 300 km. In the acoustic-air conductor of Fig. 2, the corresponding velocity v is approximately 34,500 cm. per second at ordinary room temperature; so that the wave-length λ at an acoustic pitch of 1000 ~, is about 34.5 cm. The smooth reflecting plug P (Fig. 2) corresponds acoustically to an insulated line end B in Fig. 1. The acoustic impedance of the air-column

FIG. 2.



Acoustic air-tube conductor carrying sound waves.

AB, with the distant end free at the plug, can then be measured by a technique to be described below. It is found that the tube possesses a line angle θ vector hyperbolic radians, and also an acoustic surge impedance z_0 mechanical abohms, or dynes per cm./sec. $\angle$.

If the tube is short, and has a smooth inner wall, the acoustic angle $\theta = \theta_1 + j\theta_2$ is nearly all imaginary. That is, the real component θ_1 may be taken as zero to a first approximation, and

$$\theta = j\theta_2 = j\frac{L\omega}{v} \qquad j \text{ hyps.} \qquad (10)$$

where L is the length AP of the tube, in cm., v is the velocity of sound through air in cm./second, and $\omega = 2\pi f$, is the impressed angular velocity (circular radians per second) of the impressed pitch frequency $f \sim$. Consequently, holding the pitch f steadily, the line angle θ can be adjusted to any desired value in j hyperbolic radians, or real circular radians, by altering the position of the plug P. The position angle at P is very nearly $\frac{j\pi}{2}$ hyps., as in the case of the line freed at B in Fig. 1, by formula (7), so that the acoustic impedance R_{af} at A is

$$R_{af} = z_0 \tanh \delta_A = z_0 \tanh \left(j\theta_2 + \frac{j\pi}{2} \right)$$

$$= z_0 \tanh \left(\frac{jL\omega}{v} + \frac{j\pi}{2} \right) = z_0 \coth \frac{jL\omega}{v} = -jz_0 \cot \theta_2 \text{ acoustic abohms or dynes per cm./sec. } \angle \quad (11)$$

In examining this formula, in which L is supposed to be the only experimental variable, we remember that $\frac{\pi}{2}$ being one quadrant of circular arc,

$$\tanh jo = j\tan o = jo = o \quad (12)$$

$$\tanh j\frac{\pi}{2} = j\tan \frac{\pi}{2} = j\cot o = j\infty \quad (13)$$

$$\tanh j\pi = j\tan \pi = j\cot \frac{\pi}{2} = jo = o \quad (14)$$

$$\tanh jn\frac{\pi}{2} = j\tan n\frac{\pi}{2} = \pm j\infty \quad (n \text{ odd}) \quad (15)$$

$$\tanh jn\pi = j\tan n\pi = \pm jo = o \quad (n \text{ odd, even or zero}) \quad (16)$$

In words, the tanh of one, three, five or more uneven quadrants is always indefinitely great; while the tanh of zero, two, four, or more even quadrants is always indefinitely small. If then we commence in (11) with $L = o$, the acoustic impedance R_{af} will be very great. As L is increased to one-quarter wave-length, or $L = v/4f$, θ_2 becomes $j\pi/2$, δ_A becomes $j\pi$, and R_{af} becomes nearly zero. Increasing L to a half wave-length or $v/2f$, δ_A becomes $j3\pi/2$, the impedance R_{af} returns to a high value, and when L is $3\lambda/4 = 3v/(4f)$, R_{af} is again nearly zero. Proceeding in this way, the acoustic impedance of the air-tube will be small for $L = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$, etc., and large for $L = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$, etc. If, however, the length L is not a mere fraction of a wave-length, but many wave-lengths, the real part of θ or θ_1, due to friction at, or imperfect rigidity of, the tube wall, may no longer be negligible, so that

in passing from an odd to an even number of quarter wavelengths in L, the acoustic impedance will pass from a certain maximum to a certain minimum, neither of which is remote from z_0, according to the expression $z_0 \tanh \delta_A$. In other words, it is only when L is small, with a favorable construction of air-tube, that R_{af} can be expected to approach the limits infinity and zero, as we pass from an even to an odd number of quarter wavelengths in L.

COMPARISON BETWEEN ELECTRIC AND ACOUSTIC-LINE CONSTANTS.

In the electric-line case, the linear resistance r, in ohms per wire km., is rarely a negligibly small quantity; whereas in a short length of air-tube, having a smooth polished inner wall, the linear resistance is, for many purposes, negligible. The electric linear reactance is $jx = j\mathfrak{L}\omega$ ohms per km.; where $\mathfrak{L}$ is the linear inductance in henries per km. and $\omega = 2\pi f$ is the impressed angular velocity in circular radians per second. The acoustic linear reactance $jx = jm\omega = jS\rho\omega$; where m is the linear mass of air within the tube in gm. per linear cm., S is the cross-section of the tube in sq. cm. and ρ is the quiescent density of the air in gm. per c.c. At 20° C., ρ is taken as 1.189×10^{-3} gm. per c.c. under the standard pressure of 10^6 dynes per sq. cm. or 1 megabarye. Consequently the linear reactance density $jx' = j \times 1.189 \times 10^{-3}$ mechanical abohms per c.c. This is also the linear impedance density z' vector ohms per c.c., when the resistance is ignored, reference being made to a column of air within the tube 1 sq. cm. in cross-section.

The linear leakance, or leakage conductance g mhos per wire km., is often negligible in the electric case. The same appears to be true in the acoustic case. The electric linear susceptance is $jb = jc\omega = j\omega/s$ mhos per km., where c is the linear capacitance in farads per wire km., and s its reciprocal, the linear elastance. In the acoustic case, the linear elastance is $S\gamma p_0$ dynes; where p_0 is the quiescent air-pressure, and $\gamma = 1.41$, the ratio of specific heats for air. The linear susceptance is thus $jb = j\omega/(S\gamma p_0)$. Referred to unit cross-section of air-column, the linear susceptance density is $jb' = j\omega/(\gamma p_0)$ abmhos per c.c. With negligible leakance, this is also the linear admittance density y'.

In the electric case, the linear hyperbolic angle is

$$\alpha = \sqrt{x.y.} = \sqrt{(r+jx)\ (g+jb)} = \alpha_1 + j\alpha_2 \text{ hyp. radians/km.} \quad (17)$$

In the acoustic case, it is either for unit cross-section or for the entire column of air in the tube, neglecting r and g,

$$\alpha = \sqrt{x.y.} = \sqrt{z'.y'} = \sqrt{(jS\rho\omega)\left(\frac{j\omega}{S\gamma p_0}\right)}$$

$$= j\omega\sqrt{\frac{\rho}{\gamma p_0}} = j\alpha_2 \text{ hyp. radians./cm. } \angle \qquad (18)$$

At 20° C. and under standard atmospheric pressure, this is $\alpha = j\alpha_2 = j\omega \times 2.904 \times 10^{-5}$ hyps. per cm. Thus at an impressed frequency of $f = 796 \sim$, or $\omega = 5000$, $\alpha = j\alpha_2 = j$ 0.145 hyp./cm. or 0.145 circular radian per cm. $= 8.3°$ per cm. This will be the linear retardation of phase. The wave-length at 20° C. and standard pressure will be $\lambda = 2\pi/\alpha = 2.165 \times 10^5/\omega$. At $\omega = 5000$, $\lambda = 43.3$ cm. The wave velocity is $v = \omega/\alpha_2 = 1/2.904 \times 10^{-5} = 34{,}440$ cm./sec. at 20° C. and 1 megabarye. For $r = g = 0$ this wave velocity will be independent of the frequency.

The hyperbolic angle θ, for any length of L cm. of air-tube, will be $\theta = L\alpha = jL\omega \times 2.904 \times 10^{-5}$ hyps. At $\omega = 5000$, it will be $jL \times 0.145$ hyps., and will be the same for the whole tube as for a column of 1 sq. cm.

The surge impedance in the electric case is

$$z_0 = \sqrt{\frac{z}{y}} \qquad \text{ohms } \angle \qquad (19)$$

In the acoustic case, using the same formula, but neglecting losses, it is the pure resistance:

$$z_0 = \sqrt{\frac{z}{y}} = \sqrt{jS\rho\omega \cdot \frac{S\gamma p_0}{j\omega}} = S\sqrt{\rho\gamma p_0} \text{ mech. abohms} \qquad (20)$$

which increases with the cross-sectional area S of the tube. If we consider an air-column in the tube of 1 sq. cm. cross-section, the "surge impedance density" at 20° C. and standard pressure, is

$$\mathfrak{z}_0 = \sqrt{\frac{z'}{y'}} = \sqrt{\rho\gamma p_0} = \sqrt{1.189 \times 10^{-3} \times 1.41 \times 10^6}$$

$$= \sqrt{1.675 \times 10^3} = 40.9 \frac{\text{mech. abohms}}{\text{sq. cm.}} \qquad (21)$$

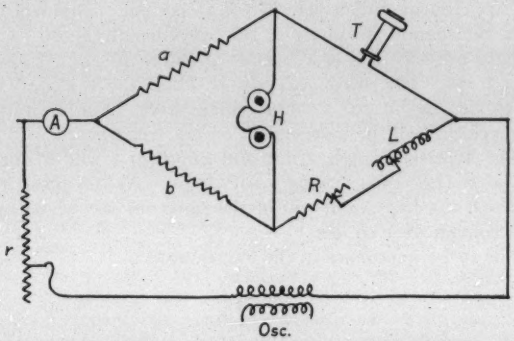
At any pressure p_0 baryes other than 10^6, and at any ordinary room temperature t other than 20° C., the surge impedance density of air becomes

$$\frac{42.4 \times \sqrt{p_0} \times 10^{-3}}{1 + 0.00183\,t} \quad \frac{\text{mech. abohms}}{\text{sq. cm.}} \qquad (22)$$

MEASUREMENT OF ACOUSTIC IMPEDANCE WITH A TELEPHONE RECEIVER.

If a telephone receiver T is connected, as in Fig. 3, with a Rayleigh Bridge, supplied by an oscillator *Osc*, which supplies a constant strength of current to the bridge at an adjustably variable frequency, the impedance of the telephone is measured in terms of the adjustable resistance R, and the adjustable inductance L. Balance can be obtained with the aid of the head telephones H. The vector impedance of the telephone is then

FIG. 3.



Rayleigh bridge for measuring telephone motional impedance.

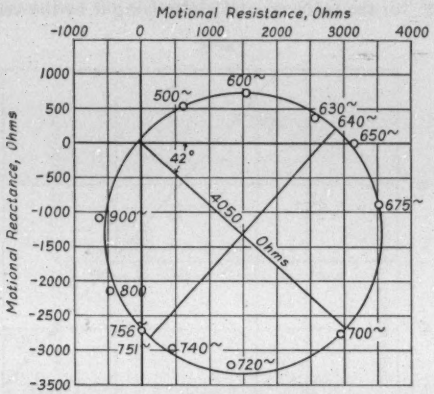
plotted, as the testing frequency is varied over a suitable range. The impedance series is next repeated with the telephone diaphragm clamped, or prevented from vibrating. The vector difference of impedance between the damped and free impedance of the telephone is then plotted. The graph is called the "motional impedance" of the telephone over the range of frequency considered; because it is the vector change in the impedance of the instrument as produced by the vibrating motion of its diaphragm. In the simplest general case, the graph of motional impedance is a close approximation to a circle [5] passing through the origin of coördinates. Fig. 4 shows the motional impedance of a certain telephone receiver having a d.c. resistance of 1150 ohms at 24° C. It will be seen that as the frequency impressed on the instrument was varied from 500 ~ to 900 ~, the motional impedance changed in

[5] Bibliography 5 and 12.

slope through nearly 180° and reached its maximum of 4050 ∇ 42° ohms = 4.050×10^{12} ∇ 42° abohms, at the diametrical frequency of 700, which was the frequency of apparent resonance in the instrument when it was supported with its aperture free in the air of the testing room.

It can be shown[6] that the motional impedance Z' of such

FIG. 4.



Motional impedance circle of a high impedance receiver (Jeanne). Measurements made with a constant a. c. current of 1.25 milliamperes through the instrument. D. c. resistance of instrument is 1150 ohms at 24°.

a receiver at a given impressed frequency, is expressed by the relation

$$Z' = \frac{A^2}{z'} \qquad \begin{matrix}\text{electric abohms } \angle \\ \text{or C.G.S. magnetic units} \\ \text{of electrical resistance}\end{matrix} \qquad (23)$$

where A is a vector constant of the instrument called the "force factor," and z' is the total mechanical impedance to the vibratory motion of the diaphragm, in mechanical abohms $\angle$. This total mechanical impedance may be expressed as

$$z' = z_d + z_v + z_1 + z_2 = z_r + z_2 \qquad \text{mech. abohms } \angle \qquad (24)$$

where z_d is the mechanical impedance in the diaphragm, z_v the virtual mechanical impedance due to vibration in the magnetic field, z_1 the acoustic impedance on the back of the diaphragm

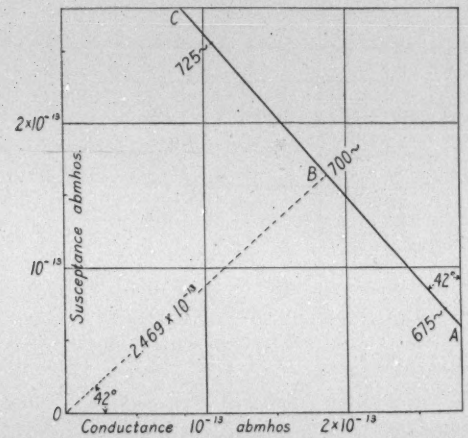
[6] Bibliography 12, page 170.

and z_2 that on the front. It becomes possible to evaluate z_2 by keeping z_1 constant, while adjustably varying z_2 in a known way. From (21)

$$z' = \frac{A^2}{Z'} = A^2 Y' \qquad \text{mech. abohms } \angle \qquad (25)$$

That is, the total mechanic impedance of the diaphragm can be found, when A and Z' are known, by taking Y' the "motional admittance" of the receiver, and multiplying it by the vector con-

FIG. 5.



Motional admittance straight line graph corresponding to the motional impedance circle of Fig. 4.

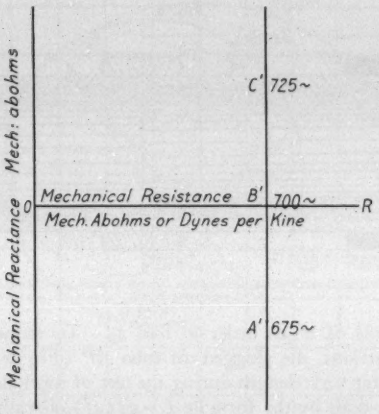
stant A^2. If the motional impedance over the given range of impressed frequency is a simple circle, as in Fig. 4, its reciprocal Y' is an infinite straight line graph ABC of admittance, as indicated in Fig. 5 at AB. Here $OB = \frac{1}{4.050 \times 10^{15} \angle 42^\circ} = 2.469 \times 10^{-13} \angle 42^\circ$ abmho, is the admittance at apparent resonance, corresponding to $4050 \angle 42^\circ$ ohms in Fig. 4. The effect of multiplying the electric admittance Y' diagram by the vector A^2 is to alter the scale to one of mechanical impedance, and to bring the straight line graph ABC vertical, as in Fig. 6. This mechanical resistance is then constant, and only the mechanical reactance varies as the

impressed frequency is varied. Part of this resistance OB' is acoustic impedance z_2.

When the motional-impedance curve Z' of the receiver departs from the simple circular form of Fig. 4, equation (25) holds; but the graph of z' departs from the simple rectilinear diagram of Fig. 6. In all cases, however, the total mechanical impedance at the diaphragm is A^2 times the vector motional admittance.

We have thus far assumed that the amplitude of vibration is

FIG. 6.



Rectilinear diagram of total mechanical impedance z' in diaphragm.

the same over the whole surface of the vibrating telephone diaphragm; whereas it is, of course, zero around the clamped edge, and a maximum over the electromagnetic poles. Consequently, when the telephone has its cap opened so as to fit into an acoustic air-tube, the acoustic impedance at the exposed surface is not $z_0 \tanh \delta_A$, but

$$z_2 = a^2 z_0 \tanh \delta_A \qquad \text{acoustic abohms } \angle \qquad (26$$

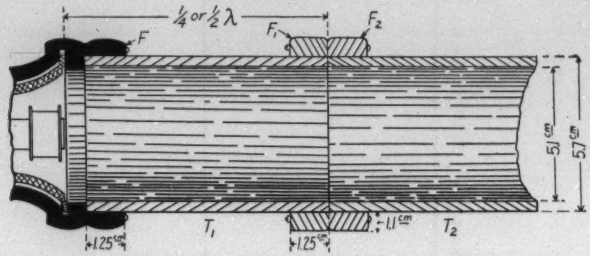
where a^2 is a real numerical coefficient, usually between 0.2 and 0.4, and equal to the mass factor of the diaphragm, *i.e.*, the ratio of the apparent concentrated mass in the diaphragm to the total actual mass within the clamping circle. The acoustic impedance which becomes effective on the diaphragm is thus much less than

that which would be developed if the diaphragm moved to and fro, like a piston, with the same amplitude everywhere.

Measurements of acoustic impedance in air-tubes have been made in this way by Prof. K. Kurokawa[7] and by Mr. P. A. Jeanne.[8]

In order to find the values of the residual acoustic impedance $z_d + z'_2 + z_1 = z_r$ in equation (21), Kurokawa devised the plan of making z_2 negligibly small and then measuring z'. The value so obtained should be the residual value to subtract from all other

FIG. 8.



Longitudinal section of telephone receiver (acoustic wave generator) and air-tube (acoustic line).

measurements of z', in order to find z_2. To reduce z_2 to very small proportions, the plugged air-tube AP (Fig. 2) is adjusted to one-quarter wave-length during the test of motional impedance. This can be done by the formula $L = v/(4f)$, already referred to, and trial can be made at this length, to see if any departure therefrom diminishes the free and motional impedances at resonance. We have already seen that a plugged quarter-wave tube, with negligible losses of sound energy, offers a minimum and sometimes negligible acoustic impedance at the generator end A.

MEASUREMENTS WITH QUARTER-WAVE AND HALF-WAVE ACOUSTIC CONDUCTING TUBES.

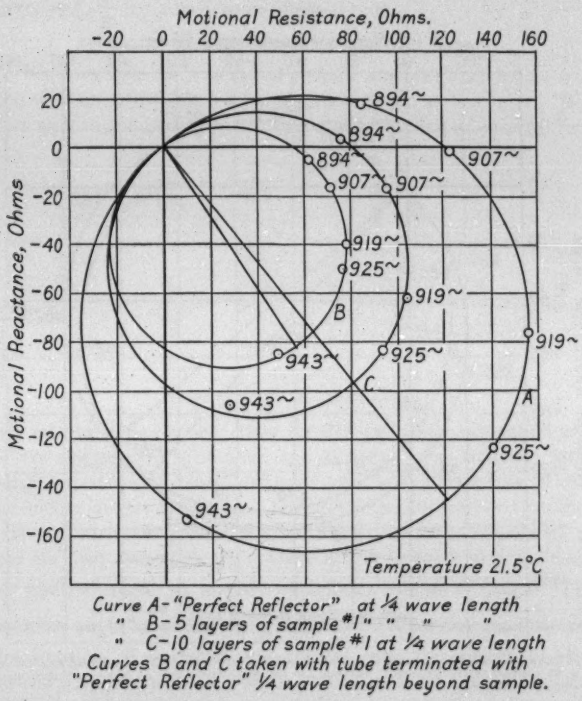
Fig. 8 is a longitudinal section of the tube and telephone used by Jeanne, and differs only in minor details from the apparatus of Kurokawa. The cap of the telephone is so modified in form as to allow the fibre tube T to fit closely with and into the instru-

[7] Bibliography 10 and 14.
[8] Bibliography 15.

ment. The tube TT could be made of half or quarter wave-length, at will, by inserting a smooth-faced plug into the open end up to the proper point.

Circle A (Fig. 10) is the motional-impedance diagram of the

FIG. 10.

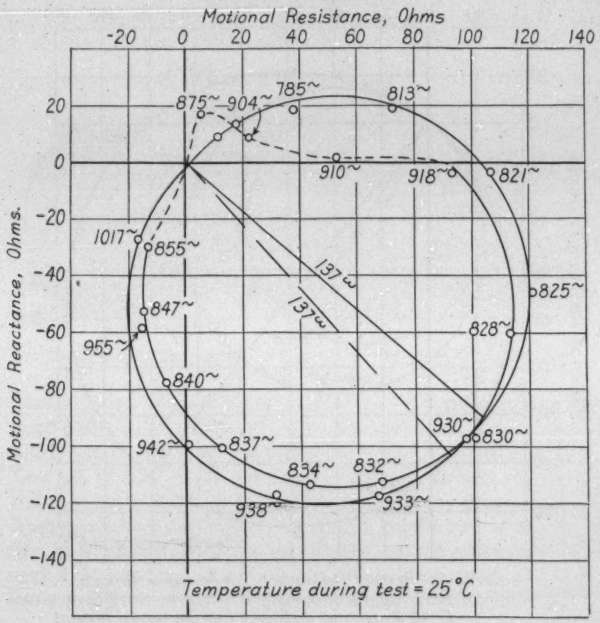


Motional impedance circles. Receiver working into fibre tube with septums of sample No. 1 located one-quarter wave-length from the diaphragm at 919 cycles.

telephone plus quarter-wave tube. The diametral motional impedance is approximately 188 ohms or 1.88×10^{11} abohms, and the corresponding motional admittance is $1/(1.88 \times 10^{11})$ or $Y' = 0.533 \times 10^{-11}$ abohms, which measures the total mechanical

impedance. According to the theory, this would be the Y' corresponding to the residual mechanical impedance $z_d + z_v + z_1$, that due to the air-column z_2 being negligibly small. Fig. 11 shows the corresponding motional impedance with the tube set at half wave-length. At 830 ~ the motional impedance is 137 ohms

FIG. 11.



Motional impedance diagram of a receiver working into a fibre tube of air with a reflecting plug at approximately half-wave distance for the pitch-frequency of 880 ~. Approximate double-circle diagram.

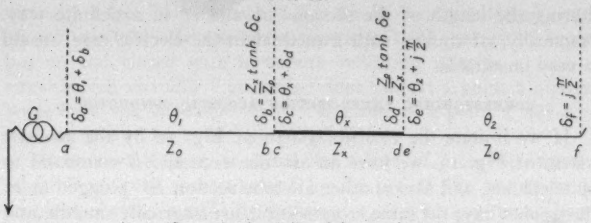
nearly on a diameter of the initial circle. At 930 ~ it is again 137 ohms, nearly on a diameter of the final circle; but at or near 880 ~, the motional impedance Z' has shrunk nearly to zero (with some uncertainty of measurement at this low value) showing that the motional admittance Y' is very large, and the acoustic impedance z_2 correspondingly very large, by (11). The motional

impedance circle of Fig. 10 has been changed to the pair of circles of Fig. 11, with nearly zero motional impedance at the half-wave pitch 880 ~ . Since z_r should be the same in both measurements, the increase may be attributed to the change in δ_A, the position angle at A, which, we have seen, is $j\pi$ for a quarter-wave, and $j\frac{3\pi}{2}$ for a half-wave plugged tube.

PROBLEM OF THE COMPOSITE ELECTRIC LINE.

We have thus far considered a single uniform electric-line section with an adjustable load at the motor end, as in Fig. 1. We have now to consider very briefly a composite line of three differ-

FIG. 12.



Composite electric line of three sections.

ent sections as in Fig. 12. Here the first and third sections *ab* and *ef* are supposed to be similar, say of aerial wire, with the same surge impedance z_0 but of different lengths and therefore of different line angles θ_1 and θ_2. The middle section *cd* is of another type, say an underground line, with its own surge impedance z_x and its own line angle θ_x. Then if the motor end f is freed, its position angle, as already explained, will be $\frac{j\pi}{2}$ hyps. The position angle at e will be $\delta_e = \theta_2 + \frac{j\pi}{2}$. Whatever may be the distributions of potential and current over the section *ef*, the potential at any point p will be proportional to the sine of the position angle, or to $\sinh \delta_p$, and the current to the cosine or to $\cosh \delta_p$. The impedance at and beyond e will be $z_0 \tanh \delta_e = z_0 \tanh \left(\theta_2 + \frac{j\pi}{2}\right) = z_0 \coth \theta_2$. Let this impedance be σ ohms $\angle$. Then the section *cd* ends at d in the load σ_2. The position angle at d, or δ_d, will not be the same as the position angle δ_e, when viewed from the section *cd*. It will be defined by (3) and (4), *i.e.*,

$\delta_d = \tanh^{-1}(\sigma_2/z_x)$. The position angle at c will then be $\theta_x + \delta_d$. Similarly, taking the section ab, the impedance at c, namely, $z_x \tanh \delta_c$, constitutes a load σ_1 at b, such that $\delta_b = \tanh^{-1}(\sigma_1/z_0)$ and the position angle at a is finally $\theta + \delta_b$. The impedance offered at a is $z_0 \tanh \delta_a$.

There are thus two abrupt discontinuities in the position angle, one at the junction d–e, and the other at the junction b–c; but there is no discontinuity in either the voltages or the current at these junctions.

It would be theoretically possible to determine the characteristics of the middle section cd (θ_x and z_x) by making successive measurements of impedance at d, keeping the end f freed, but altering the length of the sections ab and ef in a definite way. Practically, of course, such a method in the electric case, would be very unsuitable.

CORRESPONDING THREE-SECTION ACOUSTIC CONDUCTOR.

If we imitate the electric system of Fig. 12 by the acoustic system of Fig. 13, we have an air-tube section AB connected to the telephone, and also another air-tube section EF plugged at F. These tubes have the same cross-section, are internally smooth, and have the same surge impedance z_0. Their line angles θ_1 and θ_2 are supposed to be substantially devoid of real components, or to be purely imaginary. Between the two tubes is clamped a segment CD, of cloth or other material whose acoustic properties are to be determined. It is reasonably assumed that this segment will have a certain line angle θ_x and surge impedance z_x. If these two quantities can be measured, the acoustic properties of that particular septum, reflection coefficient, absorption coefficient, transmission coefficient, and velocity of propagation, for a single incident sound wave, become known. The plan is then to measure the acoustic impedance R_a at A, of the three-section acoustic conductor, making such simple and convenient changes in θ_1 and θ_2 as alterations in tube length will readily permit.

In the composite electric-line case, of Fig. 12, the section lengths might be many kilometres in length. In the corresponding acoustic system of Fig. 13, the first and third sections would be a few centimetres or decimetres long, and the middle section a few millimetres. Otherwise the two systems correspond.

TECHNIQUE FOR MEASURING θ_x AND z_x OF A SEPTUM OF SOUND-CONDUCTING MATERIAL.

It will be remembered that a smooth plugged quarter-wave tube offers an angle θ of approximately $0 + j\pi$ hyps. at its mouth and that the acoustic impedance of such a tube at that end, $z_0 \tanh j\pi$, falls to a low value, or approximates to zero. Such a tube is therefore approximately equivalent to a ground connection at that mouth point. On the other hand, a smooth-plugged tube of either half-wave or zero length, offers very large acoustic impedance $z_0 \coth j\pi$, or $z_0 \coth 0$, at its mouth. Such a tube is thus nearly equivalent to an insulated or free end at that mouth point. Consequently, in Fig. 13, if $EF = \lambda/4$ for the impressed frequency, we virtually "ground" the system at E; whereas if $EF = \lambda/2$, we virtually "free" the system at E. An open or unplugged end in an acoustic tube offers acoustic impedance to the escape of sound; but an end closed with a smooth reflecting plug in a tube of smooth wall virtually "grounds" that tube at a point a quarter-wave back and virtually "frees" it, either at a point a half-wave back or at zero back, *i.e.*, the plug itself.

It remains then so to select the values of θ in Fig. 13, that when D is freed and grounded by the use of the proper length of plugged tube in EF, the values of z_x and θ_x may be evaluated from the measurements of total acoustic impedance.

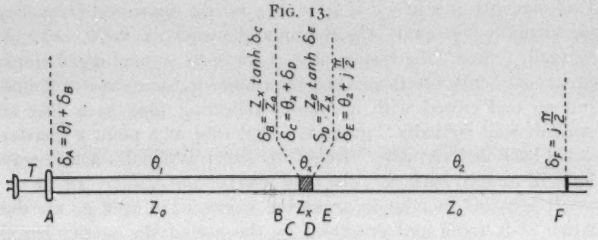
If we follow the process of attaching position angles illustrated in Fig. 12, we arrive at the results in the following table:

Table for Auxiliary Tube Lengths L_1 and L_2.

I Condition.	II L_1	III θ_1	IV$_1$ θ_x	V L_2	VI θ_2	VII $\tanh \delta_A$	VIII $Z_A - z_r$	IX Y_1
1	$\frac{\lambda}{4}$	$j\frac{\pi}{2}$	abs.	abs.	0	0	$z_0 \tanh j\pi = 0$	$Y_1 = \frac{1}{A^2}\, z_r$
2	$\frac{\lambda}{4}$	$j\frac{\pi}{2}$	θ_x	0 or $\frac{\lambda}{2}$	0 or $j\pi$	$\frac{z_0}{z_x} \tanh \theta_x$	$\frac{a^2 z_0}{z_x} \tanh \theta_x$	$Y_2 = \frac{1}{A^2}\left(z_r + \frac{a^2 z_0^2}{z_x} \tanh \theta_x\right)$
3	$\frac{\lambda}{4}$	$j\frac{\pi}{2}$	θ_x	$\frac{\lambda}{4}$	$j\frac{\pi}{2}$	$\frac{z_0}{z_x} \coth \theta_x$	$\frac{a^2 z_0^2}{z_x} \coth \theta_x$	$Y_3 = \frac{1}{A^2}\left(z_r + \frac{a^2 z_0^2}{z_x} \coth \theta_3\right)$
4	$\frac{\lambda}{2}$	$j\pi$	θ_x	0 or $\frac{\lambda}{2}$	0 or $j\pi$	$\frac{z_x}{z_0} \coth \theta_x$	$a^2 z_x \coth \theta_x$	$Y_4 = \frac{1}{A^2}\left(z_r + a^2 z_x \coth \theta_x\right)$
5	$\frac{\lambda}{2}$	$j\pi$	θ_x	$\frac{\lambda}{4}$	$j\frac{\pi}{2}$	$\frac{z_x}{z_0} \tanh \theta_x$	$a^2 z_x \tanh \theta_x$	$Y_5 = \frac{1}{A^2}\left(z_r + a^2 z_x \tanh \theta_x\right)$

Here columns II and V give the required lengths L_1 and L_2 of tube sections AB and EF (Fig. 13) in terms of the impressed air wave-length, λ. Columns III and VI give the corresponding hyperbolic angles θ_1 and θ_2, assuming negligible losses. Column VII gives the tangent of the position angle δ_A at A, and column VIII the acoustic impedance of the system beyond the point A. Finally column IX gives the motional admittance Y' of the telephone receiver with the attached tube system. Assuming that the vector force factor A of the receiver has been independently measured, and also the mass factor a of the diaphragm, there

FIG. 13.



Composite acoustic tube line of three sections.

remain four unknown quantities in the system, *viz.*, z_r the residual internal mechanical impedance, z_0 the surge impedance of the air-tube, z_x the surge impedance of the septum material, and θ_x the complex angle of the septum, which will, in general, have both a real and an imaginary component. If z_0 is calculated from the known acoustic properties of air, and the cross-sectional area of the tube, there are three unknowns, z_r, z_x, and θ_x. Three out of the five sets of measurements given in the table are theoretically sufficient for the evaluation of these three unknowns. Kurokawa, who first published this composite-conductor method,[9] uses formulas 1, 2 and 3 for Y_1, Y_2 and Y_3.

The solution of 1, 2 and 3 is

$$\tanh \theta_x = \sqrt{\frac{Y_2 - Y_1}{Y_3 - Y_1}} \qquad \text{numeric } \angle \qquad (27)$$

and

$$z_x = \frac{a^2 z_0}{A^2} \frac{1}{\sqrt{(Y_2 - Y_1)(Y_3 - Y_1)}} \qquad \text{acoustic abohms } \angle \qquad (28)$$

[9] Bibliography 14.

Jeanne also used both triple sets of formulas, the solution of 1, 4 and 5 being

$$\coth \theta_x = \sqrt{\frac{Y_4 - Y_1}{Y_5 - Y_1}} \qquad \text{numeric } \angle \qquad (29)$$

$$z_x = \frac{A^2}{a^2} \sqrt{(Y_4 - Y_1)(Y_5 - Y_1)} \qquad \text{acoustic abohms} \qquad (30)$$

Knowing the complex value of $\tanh \theta_2$, θ_x is at once obtainable from charts or tables. If the thickness of the test septum is l cm., the linear hyperbolic angle

$$\alpha_x = \frac{\theta}{l} = \alpha_1 + j\alpha_2 \qquad \text{hyps./cm. } \angle \qquad (31)$$

The wave-length λ_x in the material is then

$$\lambda_x = \frac{2\pi}{\alpha_2} \qquad \text{cm.} \qquad (32)$$

and the group velocity v of wave transmission through the material is

$$v_x = f\lambda_x = \frac{\omega}{\alpha_2} \qquad \text{cm./sec.} \qquad (33)$$

REMARKS ON THE TECHNIQUE.

It should be remarked that thus far the method has only been employed on samples of cotton and woollen cloths, which probably conduct sound to a great extent through the air-channels they contain, rather than through the material itself considered *in vacuo*. Moreover, it is assumed that the septa of cloth remained quiescent during the tests; whereas it is quite likely that they were subject to some bodily vibration, like that of a diaphragm. The numerical results thus far obtained at the Massachusetts Institute of Technology are not sufficiently reliable for presentation; but they indicate that this method of acoustic research is well worthy of being extended. Kurokawa's paper gives certain rough and preliminary results for a sample of woollen cloth 1 mm. thick, used as a septum for an air-tube having a cross-sectional area 9.8 sq. cm.

LIST OF SYMBOLS EMPLOYED.

A	force factor of a telephone receiver (rms. dynes per abampere ∠).
a^2	mass factor of a telephone receiver diaphragm (numeric).
$\alpha = \alpha_1 + j\alpha_2$	linear hyp. angle of a conductor (hyps. per km. or per cm. ∠).
α	linear hyp. angle of acoustic material tested (hyps. per cm. ∠).
b	linear susceptance of a conductor (mhos/km.).
c	linear capacitance of a conductor (farads/km.).
$\delta_A, \delta_B, \delta_C$	hyperbolic position angles along a composite conductor (hyps. ∠).
E	alternating electromotive force (rms. volts).
$\varepsilon =$	2.71828 (Napierian base).
f	frequency (cycles per second).
g	linear conductance of a conductor (mhos/km. or abmhos/cm.).
$\gamma = 1.41$	ratio of specific heats for air (numeric).
$\theta = \theta_1 + j\theta_2$	angle of a line or section of conductor (hyps. ∠).
θ_1, θ_2	section angles of a composite conductor (hyps. ∠).
θ_x	angle of acoustic material under test (hyps. ∠).
I_A	alternating-current strength at generator end of a line (rms. amperes).
$j = \sqrt{-1}$	
L	length of a conductor (km. or cm.).
$\mathfrak{L}$	inductance for balance (henries).
l	length of acoustic material septum under test (cm.).
λ	wave-length on conductor (km. or cm.).
λ_x	wave-length on acoustic material (cm.).
m	linear mass of air in a tube conductor (gm. per linear cm.).
p_0	quiescent hydrostatic pressure in air (baryes or dynes/sq. cm.).
$\pi = 3.13159$	
R_{af}, R_{ag}	impedances offered at generating end of a conductor, when the motor end is freed and grounded, respectively (ohms ∠).
ρ	density of air in sound conductor (gm./c.c.).
σ	load at motor end of a conductor (ohms ∠ or abohms ∠).
S	cross-sectional area of a tube conductor (sq. cm.).
s	linear elastance of a conductor (daraf-km. or abdaraf-cm.).
t	air temperature in degrees C. not far from 20° C.
v	velocity of wave transmission along conductor (km./sec. or cm./sec.).
x	linear reactance of a conductor (ohms/km. or mech. abohms/cm.).
x'	linear reactance density of an acoustic conductor (mech. abohms/c.c.).
y	linear admittance of a conductor (mhos/km. or mech. abmhos/cm.).
y'	linear admittance density of an acoustic conductor (mech. abmhos/c.c.).
$Y' = 1/Z'$	motional admittance of a telephone receiver (abmhos ∠).

Y_1, Y_2, Y_3	motional admittances under varied conditions of acoustic system (abmhos $\angle$).
Z_A	impedance offered by a conductor at its generator end (ohms $\angle$).
Z'	motional impedance of a telephone receiver (abohms $\angle$).
z_0	surge impedance of a conductor (ohms or mech. abohms $\angle$).
z'	total mechanical impedance of a receiver diaphragm (mech. abohms $\angle$); also linear impedance density of an acoustic-air conductor (mech. abohms/c.c. $\angle$).
$\mathfrak{z}_0$	surge impedance density of acoustic conductor (mech. abohms/sq. cm.); also linear impedance density of an acoustic conductor (mech. abohms/c.c.).
z_d, z_v	internal parts of mechanical impedance in diaphragm (mech. abohms $\angle$).
z_1, z_2	acoustic impedances at inner and outer surfaces of diaphragm (mech. abohms $\angle$).
$z_r = z_d + z_v + z_1$	residual mech. impedance of a diaphragm after deducting acoustic impedance on outer surface (mech. abohms $\angle$).
z_x	surge impedance of acoustic septum material (mech. abohms $\angle$).
$\omega = 2\pi f$	impressed angular velocity (radians/sec.).
ab	prefixed to a unit represents a C.G.S. unit and in the electric system a C.G.S. magnetic unit.
hyp.	an abbreviation for hyperbolic radian.
$\angle$	sign indicating complex or plane vector quantity.

BIBLIOGRAPHY.

(WITHOUT PRETENSIONS AS TO COMPLETENESS.)

1 LORD RAYLEIGH: "Theory of Sound," Vols. I and II, Macmillan and Co., 1894.

2 H. LAMB: "The Dynamical Theory of Sound," E. Arnold, 1910.

3 A. E. KENNELLY: "Artificial Lines for Continuous Currents in the Steady State," *Proc. Am. Ac. Arts and Sc.*, **44**, No. 4, August, 1908, pp. 97–130.

4 A. E. KENNELLY: "The Application of Hyperbolic Functions to Electrical Engineering Problems," Univ. of London Press, 1911–1924, and McGraw Hill Book Co., New York.

5 A. E. KENNELLY and G. W. PIERCE: "The Impedance of Telephone Receivers as Affected by the Motion of Their Diaphragms," *Proc. Am. Ac. Arts and Sc.*, **48**, No. 6, 1912, pp. 111–151.

6 A. E. KENNELLY: "Tables of Complex Hyperbolic and Circular Functions"; also "Chart Atlas of Complex Hyperbolic and Circular Functions," Harvard Univ. Press, 1914–1924.

7 A. E. KENNELLY and H. A. AFFEL: "The Mechanics of Telephone-receiver Diaphragms as Derived from Their Motional Impedance Circles," *Proc. Am. Ac. Arts and Sc.*, **51**, No. 8, November, 1915.

8 A. E. KENNELLY and H. O. TAYLOR: "Some Properties of Vibrating Telephone Diaphragms," *Proc. Am. Phil. Soc.*, **55**, 1916, pp. 415–460.

9 A. G. WEBSTER: "Acoustical Impedance and the Theory of Horns and of the Phonograph," *Proc. Nat. Ac. Sc.*, May, 1919, pp. 275–282.
10 A. E. KENNELLY and K. KUROKAWA: "Acoustic Impedance," *Proc. Am. Ac. Arts and Sc.*, **56**, Feb., 1921, pp. 1–42.
11 G. W. STEWART: "The Performance of Conical Horns," *Phys. Rev.*, 1920, pp. 313–326.
12 A. E. KENNELLY: "Electrical Vibration Instruments," Macmillan and Co., N. Y., 1923.
13 G. W. STEWART: "Acoustic Filters," *Phys. Rev.*, Dec., 1922, **20**, pp. 528–551.
14 K. KUROKAWA: "Measurement of Acoustic Constants of Cloth," *Jour. Ins. of Elec. Engrs.*, Japan, Feb., 1923, pp. 1–27.
15 PAUL A. JEANNE: "Acoustic Constants of Materials and Their Determination," a thesis toward the degree of S.M. in Elec. Eng. at Mass. Inst. Technology, Aug., 1923.
16 C. R. HANNA and J. SLEPIAN: "The Function and Design of Horns," *Jour. A. I. E. E.*, Mar., 1924, pp. 250–256.
17 G. K. STEWART: "Acoustic Wave Filters; Attenuation and Phase Factors," *Phys. Rev.*, **23**, April, 1924, p. 520.
18 H. B. PEACOCK: "Acoustic Wave Filters; Predicted Transmission Curves," *Phys. Rev.*, **23**, April, 1924, p. 525.
19 A. N. GOLDSMITH and J. R. MINTON: "The Performance and Theory of Loud Speaker Horns," *Proc. I. R. E.*, Aug., 1924, **12**, No. 4, pp. 423–483.

Treatment of Furnace Coal. (*The Marine Engineer and Motorship Builder*, Aug., 1925, p. 281.)—The steam-raising capacity of a boiler, whether for marine or other purposes, depends very largely not only upon the class of coal used as fuel, but to some large extent upon the method of firing and treatment of the coal before it is placed in the furnace. Where high temperatures are required, a higher degree of heat can be obtained by the use of dried rather than wet coal, as the temperature of combustion decreases with the increased percentage of moisture. For instance, a coal containing 3 per cent. moisture, as it frequently does, if passed through a drier, would give approximately a temperature of 3200° F. when burnt with 25 per cent. excess air, whereas the same coal containing 10 per cent. moisture would give only 3000° F. when burned with 25 per cent. excess air. In addition to the above, wet coal is apt to form coke in the furnace. If it is desired to obtain conditions such that the coal will ignite instantly, the moisture contents should not exceed one-half of 1 per cent. The coal on its way to the furnace after it has been dried absorbs additional moisture, and by the time it is ready to be used it may contain as much as 0.7 of the 1 per cent. moisture. Dry coal is desired because it can be more intimately mixed with air and fed more regularly to the furnace. The use of wet or damp coal makes it impossible to maintain as high a temperature in the furnace as when dry coal is used. R.

ATMOSPHERIC ELECTRIC OBSERVATIONS TAKEN AT YALE UNIVERSITY DURING THE TOTAL ECLIPSE OF JANUARY 24, 1925.

BY

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Associate Editor.

THE atmospheric electric elements measured were potential gradient and residual ionization in a closed vessel. It was felt that any possible effects of the eclipse would be more likely to show themselves in these elements than in the ionic content or the atmospheric electric conductivity as measured at the earth's surface. At any rate, any effect on conductivity or ionic content other than those arising from changes in potential gradient or extra-terrestrial penetrating radiation would probably be the result of secondary causes, such as change of humidity, atmospheric radio-active content, temperature, etc., resulting from the eclipse, and would not be of such primary interest as changes in the potential gradient and residual ionization themselves. Even such changes of the conductivity as would result from change in intensity of ion production would only show themselves with considerable lag. Thus, for example, if q is the rate of production of ions, n the number of ions per c.c., the numbers being supposed equal for both signs, as an approximation, and if α is the coefficient of recombination of ions, we have

$$\frac{dn}{dt} = q - \alpha n^2$$

which leads to

$$t = \frac{1}{2q\alpha} \log \frac{1 + n\frac{\alpha}{q}}{1 - n\frac{\alpha}{q}}$$

for a case where $n = 0$ when $t = 0$. Or, since n_0, the equilibrium value of n, is given by

$$n_0 = \frac{q}{\alpha},$$

$$t = \frac{1}{2\alpha n_0} \log \frac{1 + \frac{n}{n_0}}{1 - \frac{n}{n_0}}.$$

Thus the time for the number of ions per c.c., and consequently the conductivity to grow to half of its final value, is given by

$$t = \frac{1}{2\alpha n_0} \log_e 3.$$

Since α = about 2×10^6 for atmospheric ions, and n_0 = about 1000, we find t = 230 seconds.

Hence, if, starting with zero ions and zero q, we were suddenly to create a q such as would bring n to a final value of 1000 ions per c.c., 230 seconds would elapse before n had reached half its final value. This example will serve to indicate the lag which might be expected in the response of the conductivity to changes in the rate of production of ions.

INSTRUMENTAL ARRANGEMENTS.

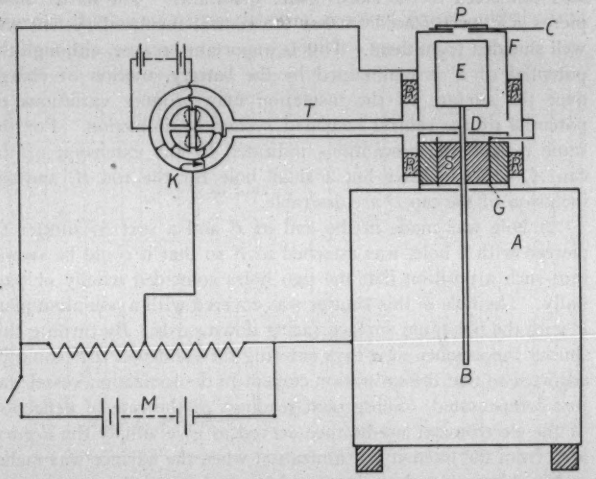
The Potential Gradient.—A water dropper was used to obtain relative values of the potential gradient. It consisted of a metal tube which extended from a window in the attic of the Sloane Physics Laboratory and which carried at its end a fork of glass with two prongs. The tube was connected to a sulphur insulated can of water placed within the building. The object of the two prongs was to save the situation in case one of the jets froze during the low temperatures which prevailed at the time of the measurement; and as an additional precaution the water which was placed in the can from time to time was heated and also rendered acid. The acid, moreover, served to increase the electrical conductivity of the water.

The insulated can was connected to the needle of a Dolezalek quadrant electrometer, to whose quadrants were connected the terminals of a battery of two small dry-cells, the mid-point of the battery being connected to the electrometer case and to a water-pipe. The electrometer was used in conjunction with a lamp and scale and the scale was subsequently calibrated. In some of the observations taken on days following the eclipse, the quadrant electrometer, which was required for other purposes, was replaced by a Wulf bifilar electroscope.

The Residual Ionization.—In order to insure a magnitude of the effect which would permit of satisfactory measurements over short periods of time, a large zinc vessel of about 300 litres capacity was used as the vessel in which the residual ionization occurred. With such a vessel, a rate of production of ions

amounting to 10 cm. per c.c. per second would give a saturation current corresponding to a movement of about nine divisions per second on an electrometer of sensitivity 1000 divisions per volt, the total capacity of the insulated system being of the order of 50 cm. Such a large rate of movement would be unsatisfactory from the point of view of the irregularity of motion which would be introduced by the inertia of the needle, although a large effect is desirable for the purpose of obtaining readings rapidly. In order to avoid difficulties resulting from rapidity of motion of the

FIG. 1.



needle, and to provide for other desirable conditions, the apparatus indicated in Fig. 1 was used.

A is the ionization chamber. *B*, the central rod, is mounted in sulphur at *S*, the plug of sulphur being divided into two parts by the guard ring *G* which was connected to the electrometer case. The rod is prolonged into a cylinder *E* which serves as the inner member of a condenser whose outer member *F* is insulated from the can.[1] The metal tube *T* which is insulated from

[1] For clearness the parts comprising the condenser *EF* and the insulating plugs are drawn larger with respect to the size of *A* than corresponds to the actual condition.

F and A is connected to the electrometer case and through it passes the wire leading to the electrometer quadrant which can be connected to or released from the case by means of the key K. The battery M of 300 volts was joined to the two ends of a megohm whose mid-point was connected to the electrometer case. In this manner, by adjusting the capacity of the condenser EF to equality with the capacity of the ionization vessel with its central rod, it was ensured that fluctuations of the battery potential produced no effect on the potential of the central system which was connected to the electrometer quadrant.[2] The hard rubber pieces RR were placed so that the central insulated system was well shielded from them. This is important because, although the potential of F is maintained by the battery, motion of charge over the surface of the insulation may produce variations of potential of the central insulated system by induction. For the same reason, the precautions indicated by the extension of the can A, so as to leave but a small hole for the rod B, and the inclusion of the cap D are desirable.

A hole was made in the end of F and a sort of shutter C, pierced with a hole, was attached to F so that it could be swung into such a position that the two holes coincided wholly or partially. The hole in this shutter was covered with a polonium plate P with the polonium surface facing downwards. By turning this shutter the number of α-rays entering the condenser EF could be adjusted so that the saturation current in the ionization vessel was just compensated. Subsequent readings of the rate of deflection of the electrometer needle then served to give simply the *departures* from the intensity of ionization when the balance was made, and in this way undesirably rapid movements of the electrometer needle were avoided. It was obviously possible to standardize the instrument in such a way that these variations in the intensity of ionization as recorded by the rate of electrometer deflection could be expressed in terms of the compensated portion of the intensity of the ionization. The regularity of the observations was such as to render it worth while to correct for the decay of the polonium in observations taken on different days.

The whole apparatus apart from the battery M was insulated on sulphur, a precaution which is desirable to provide for the

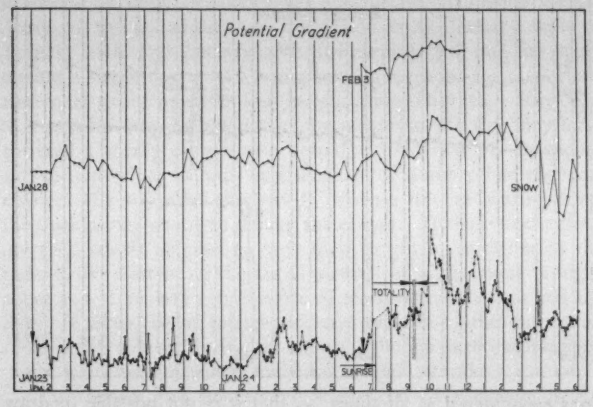
[2] W. F. G. SWANN, *Phys. Rev.*, 17, pp. 240–242, 1921.

most efficient operation of the scheme for compensating for fluctuation of the battery potential. The usual advice that the shields of an electrostatic system should be *grounded* is one of the most misleading fictions of electrostatic science, and is frequently the source of much trouble.[3] All that is necessary is that the parts in question should be connected together.

RESULTS.

Fig. 2 represents the results for the potential gradient, and Fig. 3 those for the residual ionization, relative values being given. The lowest of the potential gradient graphs and the mid-

FIG. 2.



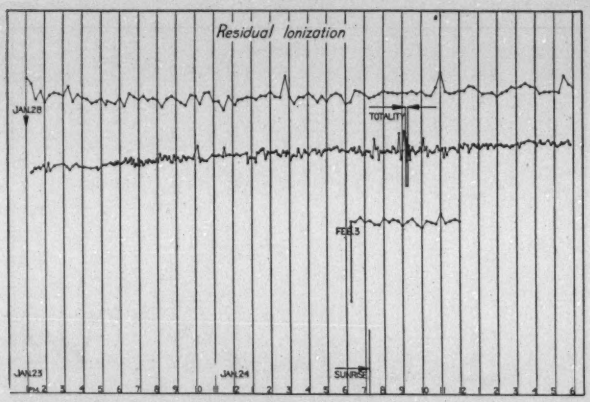
dle one of the residual ionization graphs correspond to observations covering the period from 1 P.M., January 23rd, to 6 P.M., January 24th, and include the period of the eclipse which is designated. In each of these curves the period of totality is indicated by two parallel vertical lines with arrows pointing to them and labelled "totality." A set of observations was taken on January 28th, a day on which the weather conditions approximated to those of January 24th; and, on February 3rd, observations were taken over a period of time which covered the period

[3] If anything is to be grounded, the best part to ground is the mid-point of the battery.

of the eclipse at New Haven on January 24th. In order to avoid confusion the curves of January 28th and February 3rd have been drawn with origins different from those for January 24th. In order to reduce them to the same origin as those for January 24th, they should be moved up or down, as the case may be, to the extent indicated in magnitude and direction by the vertical arrow at the extreme left of each graph, the length of the arrow being taken between the unmarked end and the tip of its head.

Such fluctuations of the potential gradient as may have taken place during the eclipse are no larger than the ordinary fluctua-

FIG. 3.



tions encountered at all times, so that it is not possible to draw any very definite conclusions from them.

The rise of potential gradient following sunrise seems to be interrupted by a tendency towards depression of the curve during the period of the eclipse. This phenomenon has been noted in former eclipse observations, and a possible explanation of it has been considered by the writer.[4] It must be admitted, however, that the form of the curve for January 28th, over the same time interval, shows features somewhat similar though less marked than those for January 24th.

[4] "The Sun's Influence on the Diurnal Variation of the Atmospheric Potential Gradient," *Monthly Weather Review*, **47**, pp. 453–456, 1919.

The most characteristic feature of the residual ionization curve, for January 24th, is the marked fluctuation of the ionization during, just after, and more particularly just before totality. One naturally wishes that these observations could be repeated, although there seems to be no instrumental reason for doubting the reality of the effect. The observations were taken in a manner exactly similar to that adopted in all the other observations and the general steadiness of the whole series of observations as evidenced by the curve is such as to lend considerable confidence to these results. It is perhaps worth while to call attention to a somewhat similar irregularity shown just after sunrise. I do not wish to attach too great weight to the continual steady increase in ionization shown by the curves. Allowance was made for the decay of the polonium over the period from January 24th to January 28th, and January 28th to February 3rd, and the decay during a single day would be too small to account for a spurious rise equal to the apparent rise in ionization observed. However, it is possible that the amount of the polonium effective may have suffered a steady change as a result of oxidation of the copper surface on which it was deposited or of some other chemical change. In any case, this steady increase has no bearing upon the fluctuations observed during the eclipse. If these fluctuations are real, one is tempted to seek their origin in variation of the radio-active content of the air as a result of air motions brought about as a secondary consequence of temperature changes and the like. It would be of interest to have records of the absorption radio-active content during an eclipse, although such observations might be misleading unless extended to high altitudes.

The curves give no marked suggestion of a change in the residual ionization from night to day, so that one could hardly expect that any direct radiation travelling from the sun straight through our atmosphere to the earth's surface was involved in the ionization. Thus, we have no right to expect during the eclipse any diminution in the residual ionization of the kind which would follow from the existence of such a radiation. Indeed, such a diminution would follow the regular course of the light intensity while the phenomenon actually observed, partaking as it did of a fluctuating character, was quite different from this. The practical equality of the day and night residual ionization suggests that if the sun is responsible for the effect at all, it must

operate through an agency which enters the earth's atmosphere more or less uniformly from all sides; and, in this connection one is naturally led to think of a corpuscular radiation controlled in its direction by the earth's magnetic field, and operating either directly or through the agency of hard X-rays or γ-rays produced when the corpuscles strike the higher regions of the atmosphere. Under such conditions one would expect that the effect of the moon would be to cause a change in the total radiation received by the earth as a whole rather than in the radiation received at any point. For this reason, observations of potential gradient and residual ionization observations taken on days preceding and following the eclipse are of interest. Insofar as there is any difference at all, the residual ionization observations for the day of the eclipse are less than, and the potential gradient observations greater than, those taken on January 28th and February 3rd, the time interval being the same in all cases and corresponding to the period during which the moon's shadow fell upon the earth on the day of the eclipse. Quite apart from the possibility of a slow, spurious increase of the apparent residual ionization already referred to, it is naturally out of the question to draw any far-reaching conclusions on this score, however, at any rate until more observations of a similar kind are available.

In connection with the tedious work of taking eye observations of the instruments on the three days cited above, I take pleasure in expressing my appreciation of the services of Mr. James Broxon and of the following graduate students in the Department of Physics: Miss F. E. Colpitts, Miss M. Goyer, Messrs. B. T. Barnes, C. del Rosario, J. E. Henderson, S. Jacobsohn, T. H. Johnson, C. A. Kotterman, E. O. Lawrence, T. E. W. Schumann.

Platinum. (*Compressed Air Magazine*, Sept., 1925, p. 1387.)—The present position of platinum in the industrial world is a peculiar one. It has always been a rare metal, and our modern inventors have made it quite an essential, in perhaps minute quantities, in many lines of industry. The demand thus created has sent it to the top of precious metals—its recent price ranging at four or five times that of gold. The enormous price has created an abnormal demand among persons fond of display. The United States now "consumes" more platinum than all the other countries of the world combined; and by far the greatest amount of the metal goes into jewelry. For this purpose the demand increased from 82,000 ounces in 1819 to 106,000 ounces in 1923, while the total world output in 1924 was only about 88,000 ounces. R.

ATMOSPHERIC ELECTRIC OBSERVATIONS TAKEN AT THE UNIVERSITY OF CHICAGO DURING THE TOTAL ECLIPSE OF JANUARY 24, 1925.*

BY

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Ryerson Laboratory, University of Chicago.

OBSERVATIONS of the potential gradient and the residual ionization in a closed vessel were made at the suggestion of Prof. W. F. G. Swann. For the potential gradient a radio-active collector was used. It was connected to the needle of a Dolezalek electrometer, one quadrant of which was maintained at a fixed potential above the other which was connected to the case. Two separate outfits of this kind were arranged; one was used in conjunction with a photographic recording drum, and the other was employed for eye observations.

The residual ionization was observed by a scheme essentially similar to that described in Professor Swann's paper, p. 490, of this number of the JOURNAL. The vessel was, however, much smaller, its volume was five litres, and no attempt was made to balance the main part of the ionization by means of a polonium plate. However, a null method of taking readings was used, essentially the same as that due to Townsend. The residual ionization apparatus had been in use in connection with another investigation and was used for the eclipse observations practically without change.

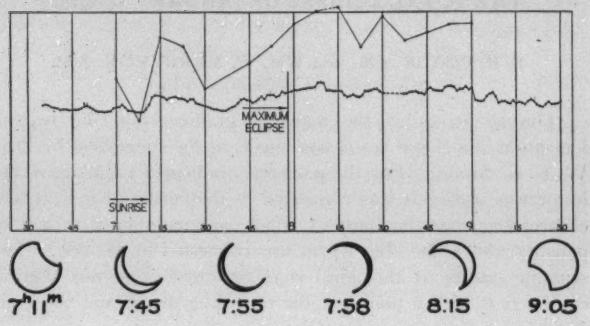
RESULTS.

The results for the potential gradient observations are represented in Figs. 1 and 2, and those of the residual ionization are shown in Fig. 3. Relative values are employed in each case. The large numbers in the abscissæ of Figs. 1 and 3 represent hours A.M. and the smaller numbers the minutes of time. Fig. 2 is a copy of the photographic trace record of the potential gradient. The time extends over twenty-four hours and covers the period of the eclipse. For reference, approximate drawings of the visible part of the sun's disc are given for the period of the eclipse

* Communicated by Dr. W. F. G. Swann, Associate Editor.

at the foot of each figure. Of the two potential gradient curves in Fig. 1, the one with a large number of points represents eye readings, while the other represents, for the period of the eclipse,

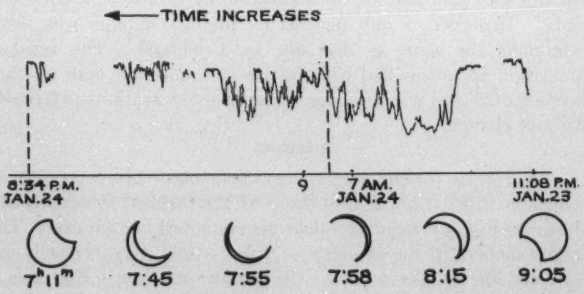
Fig. 1.



observations copied from the photographic trace record, *viz.*, Fig. 2.

The two potential gradient curves do not agree quantitatively in their variations, possibly as a result of a non-linearity in the

Fig. 2.

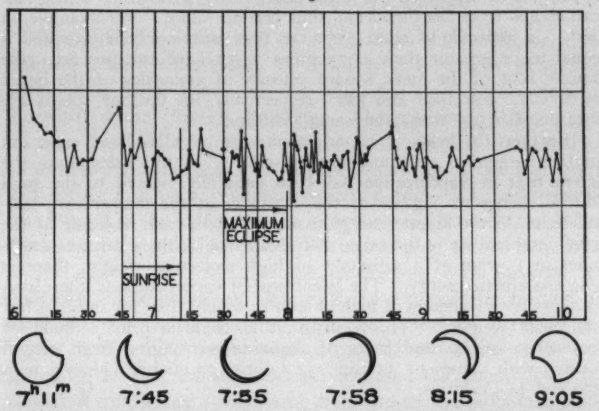


scale of one of the electrometers, or more probably as a result of the disposition of the two different collectors. The radio-active plates were fastened at the end of brass tubes, which were about one foot apart and projected horizontally out of a third-story window for about eight feet. However, the two curves do agree

qualitatively, and although they cannot be said to present any outstanding feature, yet there is a suggestion of a rise during the eclipse to a maximum which was attained about fifteen minutes after the maximum totality.

The residual ionization observations show a degree of fluctuation which is as great just after the maximum totality as it is at any other time during the period of the observations. Comparison may be made in this connection with the data obtained by Pro-

FIG. 3.



fessor Swann at New Haven with the residual ionization apparatus operated there. In the Fig. 3 here the observations before 6:15 A.M. should be regarded as preliminary to getting the apparatus into smooth running shape. However, the curve of the observations is such that it would seem unwise to draw any very definite conclusions as to the possible effect of the eclipse upon the penetrating radiation.

The Kinetic Theory of Evaporation. D. B. MACLEOD. (*Trans. Faraday Soc.*, Apr., 1925.)—"The phenomenon of evaporation from liquids has long been explained by the kinetic theory, on the assumption that various molecules are moving with different velocities. At any given temperature, the faster moving molecules tend to escape, leaving the remainder on the average slower. If they all moved with the same speed, it is fairly obvious that the liquid would

have no vapor pressure until all the molecules had sufficient speed to escape, when the liquid would explode. It is proposed to assume that the ratio of the density of the vapor above a liquid to the density of the liquid is really the ratio of the number of molecules with sufficient speed to escape from the liquid to the number with insufficient speed." These numbers of molecules are, presumably, in 1 c.c.

Applying this assumption to several organic liquids, for which Ramsay and Shields furnish the densities in both phases at several temperatures, and using Maxwell's equation for the probability of the occurrence of any speed among the molecules, the author finds it possible to determine the minimum velocity with which a molecule can escape from the liquid and thus become vapor. For example, at 20° C. a molecule of ethyl ether can thus escape only if its speed is equal to or greater than 2.673 times 3.933×10^4 cm. per sec. (the square root of the mean square velocity of molecules of the liquid at 0° C.). For 100° and 180°, respectively, the limiting speeds are 2.115 and 1.505 times the same quantity.

Further, the heats of vaporization were calculated and came out in fair agreement with measured results. "In this calculation, the latent heat of vaporization has been definitely related to the total kinetic energy of 1 gram of the substance at the temperature considered. As the kinetic energy associated with each molecule at the same temperature is the same, it follows that as there are less molecules in 1 gram of a substance of high molecular weight, there is also less kinetic energy. The latent heat of vaporization is, therefore, also less for substances of high molecular weight than for those of low molecular weight." This conclusion is confirmed by a table of molecular weights and heats of vaporization ranging from methyl alcohol with values of 32 and 232 calories to stannic chloride with 259 and 30.5 calories. G. F. S.

X-ray Crystal Analysis as an Auxiliary in Organic Chemical Research. R. ROBINSON. (*Nature*, July 11, 1925.)—"At the suggestion of Prof. W. L. Bragg, I recently sent to Dr. G. Shearer, of the Davy Faraday Research Laboratory, a series of four keto-acids which had been synthesized by my wife." Doctor Shearer did not know what they were, but the series to which they belonged was told to him. From crystal analysis of minute quantities of the acids he deduced the correct number of carbon atoms in each of the four types of molecules and, further, concluded that the carbonyl groups occupied .52, .67, .55, and .50, respectively, of the entire lengths of the molecules. The theoretical values for these fractions of the length are .54, .65, .50, and .48. "This outcome, considering the difficulties, is remarkable, and this is surely the most noteworthy invasion which the physicist has yet made of the domain of the purely structural organic chemist." The author thinks that such work as Doctor Shearer's will find application in biochemistry with the normal-chain unsaturated acids. G. F. S.

OBSERVATIONS ON THE VARIATIONS OF THE POTENTIAL GRADIENT OF THE ATMOSPHERE TAKEN AT THE UNIVERSITY OF MINNESOTA DURING THE TIME OF THE ECLIPSE OF JANUARY 24, 1925.*

BY

ANTHONY ZELENY, Ph.D.

Professor of Physics at the University of Minnesota.

THE apparatus consisted of a polonium plate collector insulated by a hard rubber rod, 54 centimetres in length, extending horizontally from a wooden rod supported in a metal stand on the roof of the Physics Building. The polonium plate in this manner was held 139 cm. above the roof and was connected to an ordinary lecture room electroscope consisting of one aluminum leaf acting against a vertical conducting rod. The electroscope was located in the gallery of the lecture room about 65 cm. below the level of the polonium plate. A sulphur plug on the same level as the electroscope insulated the conducting wire through the doorway. The polonium plate and the electroscope formed an insulated conducting system, with the polonium plate at the higher level. The outer metal casing of the electroscope was grounded so that the electroscope indicated the potential of the insulated conducting system and therefore of the polonium plate. With the apparatus used, when the system was discharged, about two minutes were required for it to regain the potential of the space.

The deflections of the electroscope were read by means of a reading telescope with vernier and micrometer adjustments. The position of the telescope was shifted by means of the vernier or the micrometer so as to keep the image of the leaf on a given mark in the field of view. The deflections ranged from 0.18 mm. to 23.5 mm., representing potentials of 27 to 730 volts.

The observations were continued from midnight until 7:00 P.M. at various intervals, which for one hour during the period of the maximum part of the eclipse were one minute apart.

* Communicated by Dr. W. F. G. Swann, Associate Editor.

FIG. 1.

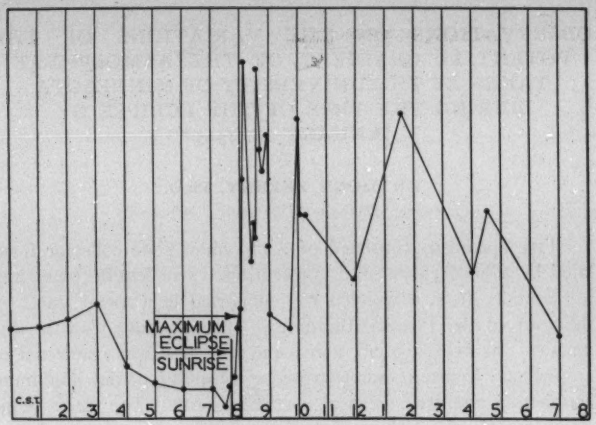
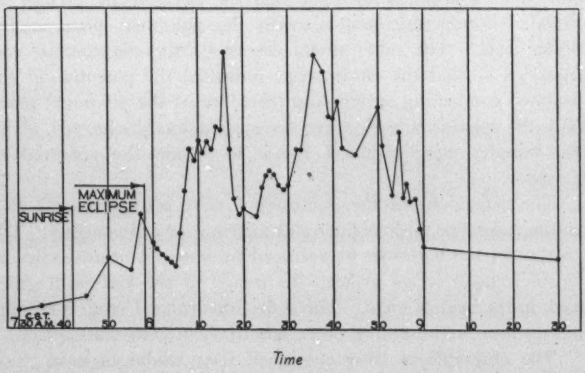


FIG. 2.



The curve of Fig. 1 shows the general nature of the variations of the potential from midnight to 10:20 A.M. with a few observations extending until 7:00 P.M. The time of sunrise

and the time of nearest approach to totality are indicated on the curve. The University is located about 138 miles from the umbra, and the maximum obscuring of the sun was about 96 per cent.

The curve of Fig. 2 gives the variations in greater detail for the region of nearest approach to totality, drawn from the same observations as the curve of Fig. 1. The scale used for plotting the potential is unaltered, while the scale for the time is magnified ten times.

The variations in the potential are so great and irregular that no definite conclusion can be drawn from the curves with regard to the effect of the eclipse on the potential gradient. Some conclusions may possibly be drawn when more similar observations are available.

The writer takes pleasure in expressing his appreciation of the services of Mr. Walter M. Nielsen and Mr. John G. Kralovec for taking the above observations for him. The observations taken by the writer personally in the region of totality at Biwabic, Minn., were interrupted at the critical time by sleet which froze on the polonium plate and the insulators, preventing the taking of observations after 6:40 A.M.

These observations were made with the hope of supplementing the work of Prof. W. F. G. Swann, of Yale University.

Production of Sodium Nitrite from Air.—Few industrial developments are of more importance in this country than the fixation of nitrogen. Several different methods have been largely exploited, but as yet no very extensive production obtained. The manufacture of sodium nitrite by the arc method is now carried out successfully by the American Nitrogen Products Company of Seattle, and the methods employed are set forth in a paper by Prof. H. K. Benson, of the Chemical Engineering Department of Washington University. This was presented at the forty-eighth meeting of the American Electrochemical Society, held in Chattanooga, Tenn., in the last week of September.

In Europe, nitrogen fixation by arc process has been developed to a total of 37,000 tons of nitrogen per year. The chief product is weak nitric acid. Aside from the Bradley-Lovejoy plant at Niagara and an experimental plant in North Carolina, the use of the arc had not been established in the United States until 1915. The plants in Norway were investigated and license arrangements made with one of the Norway patentees, but the establishment of an American plant was attained only after much experimenting.

The furnace is made of three concrete tubes, giving an inner

tubular space, within which the arc plays, and two annular spaces, through which the entering air passes in series. Thence the air passes through ports, spirally distorting the arc where the reaction occurs.

For the cooling of the gases, the ordinary fire tube water boiler is used. After cooling, these gases are conducted in iron pipes directly to the absorption towers. The latter are of wood stave construction, lined with brick, set in Portland cement and filled with the usual type of filler, a webbed tile made by a local ceramic concern. At the base of each tower is a tank containing a solution of soda ash, which is pumped continuously to the top of the tower, distributed by splash plates over the tower fillers. As this liquor descends in a thin film it comes in contact with the ascending gases and the chemical reaction resulting in the formation of sodium nitrite results.

When the soda ash liquor has attained a nearly complete reaction, it is concentrated in an evaporator to a nearly saturated solution at its boiling point, and then discharged into crystallizing tanks where the deposition of practically pure sodium nitrite takes place. The crystals of sodium nitrite are then centrifuged, dried in a rotary steam dryer and packed in paper-lined barrels.

The reaction between nitrogen and oxygen under the influence of the arc results in production of about 6 per cent. of nitric oxide, but the high temperature causes a diminution to about 2 per cent. as the gases leave the furnace. The cooling in the steam boiler causes the formation of nitrous anhydride, which by action of sodium hydroxide is converted into nitrite. Secondary reactions also occur.

H. L.

Variation of the Intensity of Sunshine from Hour to Hour. W. KARTSCHAGIN. (*Meteorologische Zeit.*, June, 1925.)—The intensity of the solar radiation was measured by its effect upon a potassium amalgam light-cell. Six different light filters, segregating as many portions of the spectrum, were in turn interposed in the path of the sunshine. The observations were conducted during the months of July and August on the roof of a three-story house in Yalta, Crimea.

For certain of the spectral intervals the intensity for the entire day was greater in August than in July, while for other intervals the opposite was true. In August the intensities in early morning and late afternoon were weaker in comparison with the intensity at noon than was the case in July. With every filter the sum of the hourly intensities from 6:30 to noon was greater than the corresponding sum from noon to 5:30. This held alike for both months.

The curves connecting the intensity with the hour are by no means the same for all spectral intervals, though their forms are in general similar. In passing from noon, either toward early morning or toward late afternoon, sunshine grows relatively richer in long wavelengths and poorer in the short ones.

G. F. S.

OBSERVATIONS ON RESIDUAL IONIZATION IN A CLOSED VESSEL, TAKEN AT BIWABIC, MINNESOTA, DURING THE SOLAR ECLIPSE OF JANUARY 24, 1925.*

BY

HENRY A. ERIKSON, Ph.D.

Professor of Physics in the University of Minnesota.

THE ionization chamber consisted of a galvanized can of about 150 litres capacity. The central electrode was a brass rod which was insulated from the can by means of a hard rubber bushing, a guard ring, and a sulphur plug through which the rod passed to a circular plate forming one side of a small air condenser. By dividing the total voltage (about 180 volts) between this air condenser and the chamber, a balance was obtained, thus eliminating the effect of a change in the battery voltage. All the voltages used were obtained from Radio "B" batteries of dry-cells. An Edelmann string electrometer was used and was adjusted to a sensitiveness of about 100 divisions for one volt. A battery giving 90 volts was connected to the plates of the electrometer. The centre of the battery was grounded through 10^5 ohms. An auxiliary potentiometer arrangement enabled the sensitiveness to be determined by applying a known voltage to the central electrode. The sensitiveness was such that the string moved ten divisions in about 25 seconds. The capacity of the system was 10 cm. The quick return of the string enabled the observer to make an observation and record the same in an interval of the order of a minute.

Observations were made on an average of twelve an hour, beginning about 1:00 A.M., January 24th, until about 6:30, when readings were obtained every one or two minutes, except for the two- or three-minute intervals required for obtaining the sensitiveness. The readings were continued for about one and one-half hours after totality, which was 1 min. 20 sec. after 8:00 A.M.

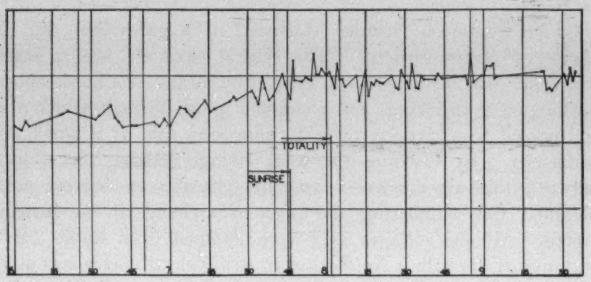
The observations from 6:00 A.M. to 9:30 A.M. are shown

* Communicated by Dr. W. F. G. Swann, Associate Editor.

graphically in Fig. 1, where the reciprocal of the time of one division deflection of the string is plotted as ordinate against the time of day.

The rise in the curve was due to a gradual increase in the sensitiveness caused by a rise in the voltage of the electrometer battery which unfortunately was short-circuited through a megohm earlier in the morning. That this was the cause is shown by the fact that the current times the sensitiveness

FIG. 1.



remained constant. The characteristics of the curve are: A somewhat increased fluctuation is shown during the eclipse and a small rise in the curve just before totality. Everything considered, however, no undue significance can be attached to the results.

This work was undertaken in order, if possible, to supplement the eclipse work of Professor Swann at the Sloane Laboratory at Yale. The observations were made in the Bangor School Building at Biwabic, Minn., and were in the path of totality. The weather was cloudy. The darkness which prevailed during totality was impressive.

I wish here to acknowledge our indebtedness to Superintendent Schweickhard for placing the Bangor School Building at our disposal and to express to Dr. and Mrs. C. W. Bray our appreciation of the many kindnesses shown us during our stay at Biwabic.

IMPACT EXPERIMENTS IN COMPOUND GASES. AMMONIA.*

BY

ALMA T. WALDIE, Ph.D.

Research Section, Randal Morgan Laboratory, University of Pennsylvania.

INTRODUCTION.

THE dynamic constants of molecules need extensive investigation beyond the present state of our knowledge before we can check effectively the various models that have been proposed to represent molecular structures. The most convincing results bearing on the constants of atoms have arisen in the studies of electron impacts designed to show points of inelastic collision, of ionization and of radiation generation. However, there are many difficulties in dealing with compound gases by impact methods caused by the altered states of the molecules and of the atoms, due to the surface effects produced by the electrodes, the chemical activity of the hot filament, to the catalyzing action of the walls of the containing vessel or of films of gas or metal adsorbed thereon, and to the effects of radiation.

These points have been considered somewhat in detail in a paper by C. B. Bazzoni and the writer of this article, published in the JOURNAL OF THE FRANKLIN INSTITUTE in January, 1924, on "Impact Effects in Nitrogen and Nitric Oxide."[1] In that paper it was shown that NO in a hot filament device is dissociated —that the O is eliminated as tungsten oxide and that all the critical points resulting from the impacts are characteristic of nitrogen only. In the present paper the investigations are extended to ammonia in an attempt to find out whether inelastic collision points determined with electrons liberated photo-electrically can be more definitely translated into molecular constants.

The following references to previous work will indicate what is already known concerning the reaction of ammonia to heat, light, and catalysis.

Action of the Tungsten Filament.—It has been shown by Langmuir[2] that dry NH_3 at low pressure is decomposed very rapidly by a tungsten filament at 1000° K.

* Communicated by Professor Bazzoni.

Photochemical Decomposition.—Kuhn [3] studied the photochemical decomposition of NH_3 at various temperatures up to 500°. He finds the speed of decomposition is only nine times as great at 500° as at 20° and that the number of quanta absorbed per molecule decomposing varies between 2.5 and 0.3 in the same interval. Arrhenius' law does not hold, which indicates there are several intermediate reactions. The reaction slows down as decomposition progresses; it is not affected by the addition of N_2, but is retarded by the addition of H_2. The following reactions are suggested:

$NH_3 + h\nu = NH_3^*$ where NH_3^* indicates an active molecule. Part of these activated molecules return to the normal state; others may decompose by either of the following:

$$NH_3^* = N + H + H_2 + 5000 \text{ calories.}$$
$$NH_3^* + NH_3 = N_2 + H_2 + H + 17{,}000 \text{ calories.}$$
$$H + NH_3 = N + 2H_2 - 39{,}000 \text{ calories.}$$

These reactions are not susceptible to experimental demonstration. They are quoted here to show the modifications of the molecules in ultra-violet light thought plausible by Kuhn. The gas is thought by him to contain during irradiation abnormal ammonia molecules and hydrogen and nitrogen, both atomic and molecular.

Synthesis through Electron Impacts.—Anderson [4] has shown that by impacts of slow-moving electrons, synthesis of ammonia sets in about 17.7 volts, and reaches a maximum at 22 volts. Storch and Olson [5] using Anderson's method, but interpreting their results somewhat differently, state that when an arc is struck in a mixture of nitrogen and hydrogen, the synthesis takes place abruptly at the following voltages: 23, 26–27, 30–31, 33–34, 37–38, 41–42, 46. They found that the arc was essential for the synthesis of the NH_3, indicating that ionized hydrogen and nitrogen are necessary. These results can not be interpreted in full in the light of our present information.

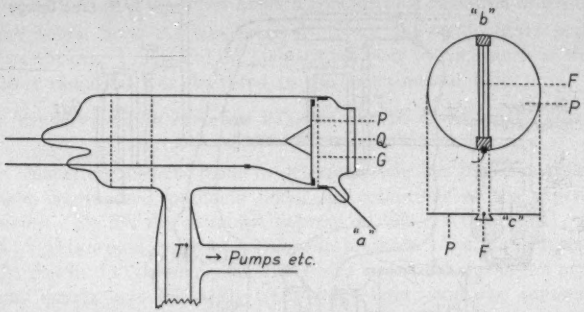
Ionization Potentials in NH_3.—Mackay [6] measured the ionization potential of ammonia by the method previously used by Hughes and Dixon, the bombarding electrons being produced by ultra-violet light. No attention was paid to the dissociation of NH_3 through the light used, and the corrections applied to the experimentally obtained curves seem somewhat doubtful. The value obtained for the ionization of NH_3 was 11.1 volts.

Catalytic Reactions between Metals and NH_3 or H or N.—

There are several kinds of reactions catalyzed by metals, such as platinum or nickel, that are of importance in the investigation of ammonia. The first of these is the well-known adsorption of hydrogen on metals,[7] and a second the oxidation [8] of NH_3 in the presence of oxygen and heated Pt or Ni. In this connection the suggestions of Sabatier [9] are of interest. He assumes that with metals like nickel, hydrogen forms an unstable hydride like NiH_2 or Ni_2H_2, which is capable of giving out atomic hydrogen, a form more active than the molecular variety.

Considerations of the facts above recorded make it evident that experiments on NH_3 carried out in a device containing

FIG. 1.



metals, hot or cold, or traversed by ultra-violet radiation need careful interpretation.

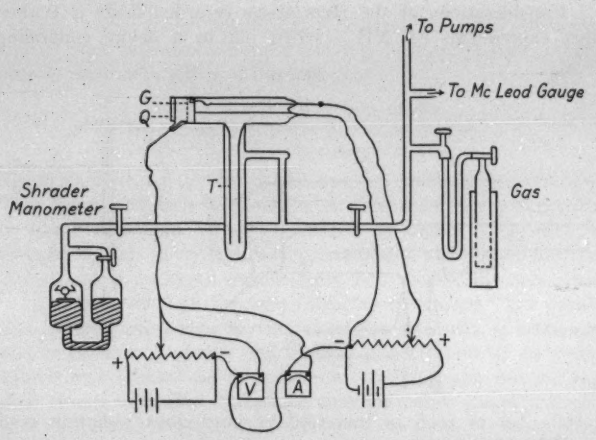
APPARATUS.

The ammonia gas used in the following experiments was generated by heating C. P. concentrated ammonia water (28 per cent. NH_3), over a water bath, drying it through lime, collecting it over mercury, and then holding it in a trap which was immersed in a mixture of NaCl and ice at $-15°$ C., or in CO_2 snow and acetone as desired. From this trap the gas could be admitted to the observation chamber when needed.

Two experimental tubes of different constructions were used. *Tube No. 1* (Fig. 1) was a two-electrode tube of pyrex glass, having a nickel plate, *P*, 4 cm. in diameter, a parallel grid, *G*, of the same diameter, 3 mm. distant from the plate and a straight

tungsten filament, F, of 3 mil wire. This filament was let into the face of the plate, being supported on lavite insulators so as to be flush with the face of the plate (Fig. 1, a, b, c). At one end of the tube was a crystal quartz window, Q, 3 mm. thick, through which light from a quartz-mercury lamp could fall upon the nickel plate for the purpose of supplying photo-electrons. The filament was connected to leads sealed into the glass. The connections used for introducing the desired pressure of gas into the tube and for

FIG. 2.



measuring the pressure changes are shown schematically in Fig. 2. The observation chamber was protected by a trap T, inserted immediately below it. Below the trap a Shrader optical lever manometer was connected to the apparatus, and also a rapid McLeod gauge. The apparatus was cleaned by an induction furnace, whereby the metal parts could be heated to a red or white heat, and held there as long as desired. The tube was exhausted by a mercury diffusion pump. In this tube the electrons used for bombarding the gas could be obtained either from the hot filament, F, or from the photo-electric disc, P, in the same plane.

Tube No. 2. This tube was a three-electrode device, having a nickel grid and plate coaxial on a straight tungsten filament of 3 mil wire. The radius of the plate was 5 mm., its height 1 cm.

The grid was placed 1 mm. from the filament. Connections in this tube were made in the usual way, so that ionization and resonance points could be determined from total current curves using the arrangement as a two-electrode device, or from partial current curves run with a retarding potential on the plate.

Ultra-violet Light.—The ultra-violet light for stimulating the photo-electron emission was obtained from a vertical Cooper-Hewitt quartz-mercury arc, operating on 220-volt circuit at 900 watts. In order that the light might reach its maximum intensity,[10] the arc was struck fifteen to twenty minutes prior to the application of the light to the ammonia gas. This arc ran hot, so that the line 2536 must have been largely extinguished by absorption. On the other hand, a considerable radiation down to and below 2000 Å. was present. Since the total quartz path was not over 3 mm. and the air-path not over 10 cm., much of the short radiation was delivered in the experimental tube.

A. EXPERIMENTS ON PRESSURE CHANGES DUE TO ULTRA-VIOLET LIGHT AND TO THE HOT FILAMENT.

Experiments were made to determine whether the ultra-violet light decomposed ammonia under the conditions of the experiments. Should the ammonia decompose, the reaction $2N.H_3 \rightarrow N_2 + 3H_2$ would cause an increase in pressure. For this purpose the gas in the Shrader manometer was irradiated through a crystal quartz window with ultra-violet light, and the pressure changes followed with a cathetometer. The Shrader manometer contained very little metal, merely the wire supports of the rotatable mirror. It was found that with an ammonia pressure of 12 mm. a delayed dissociation took place, the pressure rising 5 mm. in the course of one hour, after showing no increase in the first ten minutes. It was found, however, that when the ultra-violet light was used on the ammonia in the apparatus containing the nickel plate and grid, there was, under certain degrees of cleanness of the plate, a decrease in pressure rather than increase. This was thought to be due to the adsorption of hydrogen on the nickel plate. On occasions when a trace of air was present in the ammonia, a much more marked decrease in pressure occurred when the ultra-violet radiation was used in the tube. This decrease was ascribed to the oxidation of the ammonia under the catalytic action of the illumination on the plate. Ultra-violet

light is thus seen to activate the nickel catalyst much as heat does in the ordinary commercial application of a similar reaction to ammonia oxidation. This marked and characteristic pressure decrease was not obtained in ammonia known to be air free.

When the hot filament was used with gas pressures about .05 mm. in tube No. 1, an immediate and very considerable increase in the pressure was observed, indicating a dissociation of the gas. On account of the large volume of the gas, in the Shrader manometer and in its connecting tubes, not in contact with the filament, no quantitative estimate of the extent of the dissociation could be made. These observations demonstrate that ammonia is dissociated both by a hot filament and by ultra-violet light.

A critical consideration of the results of these experiments, in which the pressure changes could be readily and instantaneously followed in the Shrader manometer, involves the following points. It is to be observed that the dissociation by the hot filament is a contact effect taking place at the filament and that the products of dissociation will, except at very low pressures, remain near the hot filament. When, as with ammonia, the products of dissociation are nitrogen and hydrogen, the nitrogen will be removed from the vacuum by the well-known interaction of tungsten and nitrogen in an electrical field, leaving mostly hydrogen close to the filament, and undissociated gas at a distance from it provided the filament chamber is in communication with a considerable volume of gas.

Dissociation by ultra-violet light is, on the other hand, a volume effect, the affected molecules being distributed throughout the irradiated volume at moderate pressures. The evidence indicates that here, as in previously investigated radiation experiments, the light has a low efficiency in producing dissociation—only a very few of the irradiated molecules responding to the stimulation. Further, a certain induction period seems necessary. We must therefore expect to find in ammonia under ultra-violet radiation, a great number of unaffected molecules of ammonia, and a relatively smaller number of atoms or molecules of nitrogen and hydrogen. These gases at the instant of formation through dissociation are doubtless in the atomic form. In this form the hydrogen will, as is well known, be readily taken up by metals, so that in an ionization chamber lined with metal or containing nickel gauzes, we must expect a large part of the hydrogen to be

removed. Under these conditions the space will contain, for the most part, ammonia and nitrogen.

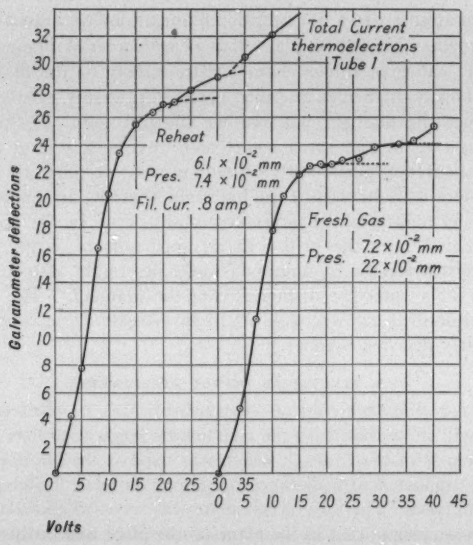
To sum up, the studies of the pressure changes here made, taken together with previously established knowledge of contact effect between metals and gases, enable us to predict that impact experiments made in a device using photo-electrically liberated electrons in an ammonia atmosphere, ought to give voltages of inelastic collision characteristic of ammonia molecules, and probably also in the presence of much metal, those characteristic of nitrogen atoms. In a hot tungsten filament device, impact experiments made with a close plate, that is with a short filament-plate distance, will give values characteristic mainly of the dissociated gases, namely, hydrogen and nitrogen, and probably mostly hydrogen, since the filament will remove the nitrogen. If, however, the design of the apparatus is such as to involve any large amount of metal surface, particularly heated surface, we must also expect a considerable reduction of the hydrogen thereby, and under these conditions undissociated molecules might occur in the bombardment region near the grid. In an apparatus of large volume, effects characteristic of ammonia molecules ought also to be obtainable at a sufficient distance from the filament. The experiments described below enable us to investigate these conclusions in a fairly decisive manner.

B. ELECTRICAL IMPACT EXPERIMENTS.

I. *Hot Filament Source.*—Apparatus No. 1.—Reference to Fig. 1 will show that this tube contained a large area plate (diameter, 4 cm.) made of nickel, which was used as the photo-electron source, supported at a distance of 3 mm. from a nickel grid of 1½-mm. mesh. The face of this plate was recessed and a tungsten filament was supported in the plane of the plate within this recess. Using the hot filament as the electron source, curves, of which those shown in Fig. 3 are examples, were obtained. These are necessarily "total current" curves, since the tube contained only two electrodes. Breaks of the current-potential curves upwards consequently indicate either ionization or radiation. In consequence of the presence of the large nickel cathode the radiation breaks might be expected to be more noticeable than usual in such tubes. These breaks are distinguishable by a curvature convex upwards. Since it was not possible to saturate the current below

20 volts, no definite evidence of breaks below this value could be obtained. There were, however, indications of a break near 18 volts. The principal breaks in the curves given are at 22 volts and near 32 volts. Of these two, the 22-volt point cannot be ascribed to hydrogen, or to nitrogen, and may safely be connected with ammonia, as a multiple of a fundamental point at 11 volts. The breaks above 30 volts may be either an upper break point of nitrogen, which previous work shows to exist at 31 volts, or the

FIG. 3.

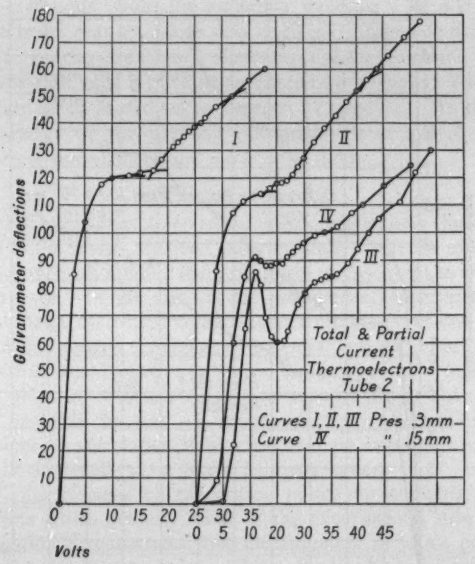


triple effect of ammonia (33 volts). The fundamental ammonia point (11 volts) cannot be seen on these curves, due to a lack of saturation in that region. The pressures used in this work are indicated on the curves. During the preformance of a run, the nickel plate in which the filament was set became very hot over a considerable portion of its area.

Apparatus No. 2.—Here the construction was of the usual coaxial filament, grid and plate type, with the grid 1 mm. from the filament and the plate 6 mm. from the filament. The curves of

Fig. 4 are examples of the results obtained with this tube. The total current curves were obtained with plate and grid connected. The grid was very close to the filament, and consequently in the volume of dissociated gas. As a consequence these curves show no trace of the characteristic ammonia effects at 11 and 22 volts. The breaks obtained [13, 16.5, (17.5), 31.5] are readily identified as belonging to either hydrogen (13.5, 16.5) or to nitrogen (17.5,

FIG. 4.



31) and probably, therefore, to both. The general aspect of the curves is, that of hydrogen curves—that is to say, the pronounced breaks—are due to hydrogen. The partial current curves, on the other hand, give definite proof of the existence of inelastic impacts at 11 and 22 volts. These impacts from the design of the apparatus occur beyond the grid, and throughout the volume of gas between the grid and the plate, namely, at a considerable distance from the filament. The volume in communication with the ex-

perimental chamber was large and filled with ammonia. We must consequently expect an enormous preponderance of undissociated molecules near the plate, and would therefore be inclined to ascribe these inelastic impacts to ammonia molecules. This, combined with other evidence, may indeed be regarded as conclusive.

II. *Photo-electric Source.*—Apparatus No. 1.—The nickel

FIG. 5.

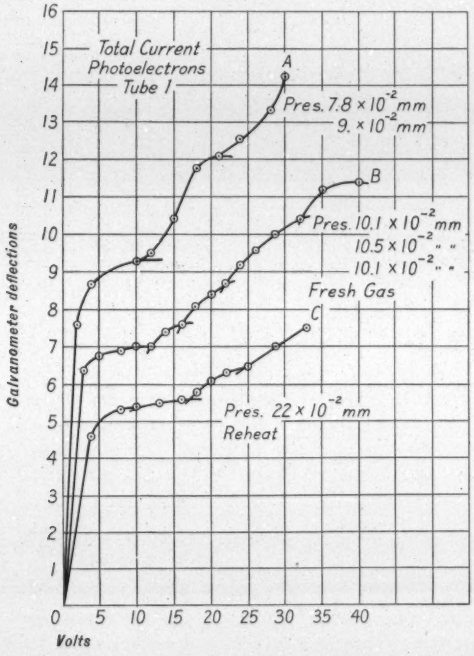


plate in this apparatus could be used as a source of photo-electrons by illuminating it with ultra-violet light through the quartz window and through the nickel grid. Under these conditions a photo-electron current of some strength could be obtained on account of the size of the disc and the power of the 220-volt light source. The photo-electrons produced had a maximum emission

energy of from .7 to .9 volt. The photo-electron current saturated at about 4 volts. The curves of Fig. 5 are illustrative of the results obtained with this arrangement. The curves shown are of two classes, *A* and *B*, developed from gas freshly introduced into the tube; *C*, obtained from old gas, which had been subjected both to treatment with the hot filament, and to extended irradiation with the ultra-violet light. Curve *A* shows ionization clearly at 11 and 22 volts. Curve *B* shows these same points, and also the 33-volt point from the ammonia molecule. In addition it shows a break near 17, which may be either nitrogen or hydrogen. Curve *C*, on the other hand, shows no definite ammonia breaks, or, at any rate, only a trace of an effect near 11 volts: The upper breaks are safely accredited to nitrogen (17 and 24). In this case the character of the gas had presumably been altered by the exposure to hot filament and ultra-violet light.

DISCUSSION OF RESULTS.

(*A*) *Concerning Corrections.*—No corrections have been applied to the results obtained experimentally, since the magnitude of the corrections renders them relatively unimportant to our present purposes. In the case of the hot-filament experiments, the most probable correction compounded from potential drop in the filament, the contact difference of potential and the initial velocity of emission would amount to several tenths of a volt. In the case of the photo-electrons, a correction somewhat less than 1 volt would probably be fair. The uncertainty in determining the corrections by calculation is so great that no important improvements in the results arise from their application.

(*B*) *Conclusions* on the critical points of ammonia and on the effects which actually occur in the experimental tube. This investigation demonstrates that the ammonia molecule possesses a critical point near 11 volts. This point has been determined in the first place by photo-electrically liberated electrons in fresh gas, the evidence indicating that the dissociation through ultra-violet light proceeds slowly and that, of the products of dissociation, the hydrogen is more or less effectively removed by the illuminated nickel gauze. It would seem from these experiments that it is safe to use ultra-violet light for impact measurements in a dissociable gas, if certain precautions are observed. The ammonia point has been determined in the second place with thermions

by arranging so that the collisions took place at some distance from the filament which was an active dissociating agent. The evidence indicates that in the immediate neighborhood of the filament, the ammonia is almost entirely dissociated, the critical points determined in that region being principally those of hydrogen.

In conclusion the writer wishes to express her appreciation of the assistance given by Professor Bazzoni, who suggested this problem and directed its progress.

REFERENCES.

[1] BAZZONI and WALDIE: JOUR. FRANK. INST., Jan., 1924.
[2] LANGMUIR: *Jour. Chem. Soc.*, **38**, p. 2279.
[3] KUHN: *Comp. Rend.*, **178**, 708–709, 1924.
[4] ANDERSON: *Zeit. f. Phys.*, 10–1–54, 4/22.
[5] STORCH and OLSON: *Jour. Am. Chem.*, 7/23, 1605–1614.
[6] MACKAY: *Phil. Mag.*, **48**, 823–835, 1923.
[7] KRACEK: *Jour. Am. Chem. Soc.*, **46**, 674–680, 1924.
[8] PERLEY and SMITH: *Ind. and Eng. Chem.*, **17**, 258, 1925.
[9] SABATIER: "La Catalyse en Chemie Organique," 60, 1920.
[10] WATTERS: *Phil. Mag.*, **48**, No. 285, 1924, p. 527.

The Isotopes of Mercury. F. W. ASTON. (*Nature*, Aug. 8, 1925.)—From the Cavendish Laboratory comes the welcome announcement that Mr. Aston has built a mass-spectrograph having twice the dispersion of his former instrument. With the new instrument he has again investigated the mass-spectrum of mercury and finds the mass numbers of its constituents to be 198 (4), 199 (5), 200 (7), 201 (3), 202 (10), and 204 (2). The bracket numbers give the relative intensity of the lines. There are in some cases faint traces of lines that might indicate additional isotopes. "These results have a direct bearing on the claims recently made that under special conditions mercury has been transmuted into gold by the addition of an electron to the nucleus. It is clear that if the gold were so formed it would have an atomic weight at least as high as 198, that is, perceptibly higher than that of ordinary gold, 197.2. A definite determination on this point would seem to provide conclusive evidence on this interesting problem." G. F. S.

AN ANALOGY BETWEEN PURE MATHEMATICS AND THE OPERATIONAL MATHEMATICS OF HEAVISIDE BY MEANS OF THE THEORY OF H-FUNCTIONS.*†

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INTRODUCTION.

THE operational mathematics of Heaviside offer a peculiarly direct and automatic method for the solution of many problems in physics and engineering. They have the disadvantage, however, that the meaning of the operations performed is not always clear, and thus a large amount of skill and experience is required in determining where this type of mathematics may be used. In Heaviside's own work, in his Electromagnetic Theory, several cases may be found where he rejects the solution of a problem obtained by one type of operator, and retains the solution obtained by another type. More recently Carson [1] has cited cases where it is necessary to add extra terms to the solution as obtained by the

* Communicated by the Author.

† In this paper an attempt is made to reconcile the operational methods of Heaviside with the methods of pure mathematics. In order to do so certain modifications in the conceptions of Function, Limit, Differential Coefficient, etc., are suggested. The new definitions are called H-functions, etc., and have the property that they give the same results as the present definitions at ordinary points, *i.e.*, points at which there are no discontinuities, etc. The difference in the results obtained by both methods is pointed out by reference to well-known cases.

The use of these H-definitions in determining Green's functions in the conduction of heat or electricity and in potential problems is shown. To do this it is necessary to extend the use of the Heaviside "unit function" with respect to time to the other coördinates, and to develop an analog of the Heaviside expansion theorem in three dimensions. The use of the Green's function and two well-known theorems to obtain solutions under arbitrary initial or boundary conditions is illustrated by two examples. The procedure in solving such problems is standardized and it appears that by its use most of the problems given in Carslaw, "Mathematical Theory of the Conduction of Heat," second edition, 1921, or Byerly, "Fourier Series and Spherical Harmonics" or other such works, may be readily solved.

[1] CARSON: *Bell System Tech. Jour.*, Nov., 1922, Vol. i, No. 2, pp. 43–55.

usual Heaviside methods. Bush [2] has also pointed out some difficulties in Heaviside's own work.

In recent years quite a number of papers have appeared on the derivation and use of Heaviside's expansion theorem. Most of these have been along the lines outlined by Heaviside himself, in that they attempt to derive the results on purely operational grounds. Dr. E. J. Berg suggested to the author that it should be possible to develop a scheme in conventional mathematics which would closely parallel the operations used in the application of the Heaviside expansion theorem. The results of the work done with this end in view were given in a paper in this JOURNAL (June, 1923) entitled "The Solution of Differential Equations by a Method Similar to Heaviside's." In that paper the "method of elimination" was given, by means of which it was possible to solve a linear differential equation of the n^{th} order with constant coefficients, by means of the solution of n linear equations with constant coefficients of the first order.

While, however, this method appeared to be fairly satisfactory, there seemed to remain certain difficulties with the solution obtained by ordinary mathematics. The most striking of these is given in the paragraph at the bottom of page 823 and top of page 824, JOUR. FRANK. INST., June, 1923, where a change in the order of operations was made which cannot be justified very well from the viewpoint of the Theory of Functions of a Real Variable. The problem was that a discontinuous function had to be differentiated at a point of discontinuity. If the ordinary methods were adopted and the function differentiated up to a point close to the discontinuity, and the solution then continued by the methods there employed, the wrong result was obtained. However, by allowing the legitimacy of the inversion of the order of operations at the point of discontinuity the correct result was derived. Thus, it would appear, the question may properly be asked, "Is it possible to arrange matters so that a discontinuous function may be differentiated at the point of discontinuity?"

As a result of a study of this and similar points, it developed that definitions might be given for the differential coefficient of a function at its points of discontinuity, provided that the concepts of function, differential coefficient and discontinuity were slightly altered. It was also found necessary to make alterations in other

[2] BUSH: *Jour. Math. and Phys.*, Vol. iii, No. 2, May, 1924.

definitions such as integration, sum of a series, etc. The new definitions, however, have the property that at what may be called the ordinary points of the function, *i.e.*, where there are no discontinuities or other peculiarities, they give the same results as the ordinary definitions. Other questions also arose such as the relation between an algebraic expression and its expansion in an infinite series, the relation between convergent, asymptotic and divergent expansions for any given function, and the field of fractional differentiation and generalized series as used by Heaviside, etc.

The purpose of this paper is to set forth some of the proposed definitions and to show how they may be used to throw light on the methods used in operational mathematics. It is not intended to attempt to deal with the questions involved exhaustively, but rather to point out the main principles which seem to be involved, and thus enlist the aid of others who may work along the same lines. The definitions as given are even to be regarded as tentative and subject to change if further investigation should show it to be desirable to do so.

It seems best, although perhaps not strictly logical, to develop in an experimental way the theory which follows. In order to avoid confusion which might arise between the new definitions and the older ones, it has been thought advisable to prefix an H to the new definitions and call them H-functions, H-differential coefficients, H-sums of series, etc. The subject itself may be called "the theory of Heaviside functions," or for brevity "the theory of H-functions."

The latter part of the paper contains examples illustrating the use of the theory in problems in mathematical physics.

H-DIFFERENTIATION.

Heaviside makes a very great use of what he calls a "unit function," which is a function whose value is zero when $t < 0$, and whose value is unity when $t > 0$. He does not state very definitely what its value is at the time $t = 0$, but as he sometimes denotes it by a symbol like $\frac{1}{2}$, it may perhaps be inferred that he considered that at the time $t = 0$ it had all values from 0 to 1. This, as it will appear, is the interpretation to be given to it, under certain conditions, which will now be investigated, by deriving such a function from another by means of H-differentiation.

It is unnecessary at the present time to define fully an H-function. In order to define H-differentiation it is sufficient to define what may be called a partial H-function and be denoted by the symbol H_P-function. The H_P-function is defined as follows: Let a and b be any two real numbers where $b > a$. If to every value of x in the continuous interval $a \leq x \leq b$,[3] there corresponds a real number y, then y is an H_P-function of x, provided that y is a continuous function of x. It will be noted that there are two points of difference between this definition and the ordinary definition of a function, (1) x must lie in a continuous interval and (2) y must be continuous. These conditions, as will be seen later, are essential. It will also appear later that there is a difficulty involved in the question of what is to be understood by the phrase "y must be continuous" but at present it will be taken to mean that y is a continuous function of x as defined in the Theory of Functions of a Real Variable.

The definition of a differential coefficient given by Hobson, Theory of Functions of a Real Variable, second edition, Vol. 1, is as follows: "If a function be defined for all points of the interval (a, b) then for the point x_1 in this interval we may regard the function $[f(x) - f(x_1)]/(x - x_1)$ as a function $F(x)$ of x_1, which is defined for all points of the interval (a, b) except for the point x_1. This function, although undefined at the point x_1, has finite or infinite functional limits at that point in accordance with the prior definitions of these limits."

"If the limits $F(x_1 + 0)$ and $F(x_1 - 0)$ both exist and have the same value either finite or $+\infty$ or $-\infty$, this value is called the differential coefficient at x_1 of the function $f(x)$."

We will accept the definition of the limits of the differential coefficient on the right and left, *i.e.*, $F(x_1 + 0)$ and $F(x_1 - 0)$, but will reject the definition in the second paragraph of the differential coefficient at the point x_1, and proceed to re-define it in the theory of H-functions as follows:

Assume first that the function or H_P-function has only one value at the point x_1 (the case where it has more than one value will be considered later). To find the H-differential coefficient of $y = f_P(x)$ at the point x_0, take two small increments in x,

[3] The symbol $\leq$ will be used throughout the paper to denote less than or equal to.

Δx_1 and Δx_2, both essentially positive numbers. Find the values of $f_P(x_0 + \Delta x_1)$ and $f_P(x_0 - \Delta x_2)$. Form the quantity

$$R = \frac{f_P(x_0 + \Delta x_1) - f_P(x_0 - \Delta x_2)}{\Delta x_1 + \Delta x_2} \tag{1}$$

Fix a definite ratio of Δx_1 to Δx_2, for instance let $\frac{\Delta x_1}{\Delta x_2} = p$, then if the value of R, as both Δx_1 and Δx_2 tend to zero, keeping p fixed, is independent of p, $f_P(x)$ has a single derivative at the point x_0. If, however, $\lim_{\Delta x_1,\, \Delta x_2 \to 0}$ R is a function of p, the H-differential coefficient of $f_P(x)$ with respect to x at the point x_0 has all the values obtained by letting p vary from 0 to ∞. Negative values of p are excluded because Δx_1 and Δx_2 are defined as positive numbers.

This definition is entirely independent of that given by Hobson and is to be used in addition to it, though it nevertheless is consistent with it. For, assuming the differential coefficient is a continuous function, this definition gives as one value of the differential coefficient

$$f'_P(x) = \lim_{\Delta x_1,\, \Delta x_2 \to 0} R \quad \text{when} \quad \frac{\Delta x_1}{\Delta x_2} \to \infty \tag{2}$$

which would correspond to the one value given by Hobson's definition

$$f'(x + 0) = \lim_{\Delta x_1,\, \Delta x_2 \to 0} R \quad \text{when} \quad \frac{\Delta x_2}{\Delta x_1} = 0 \text{ or } \Delta x_2 = 0 \tag{3}$$

with a similar equation for the lower limit. In general, using the present methods of the Theory of Functions of a Real Variable, it does not seem justifiable to assume (2) and (3) must necessarily give the same result, in fact in cases where $f_P(x_0)$ has more than one value corresponding to the given value x_0, they might not necessarily be equal, but this will be considered later.

Take as an example the function defined by

$$\begin{aligned} y &= (1 - \xi)x & x &\leq \xi \\ y &= (1 - x)\xi & x &\geq \xi \end{aligned} \tag{4}$$

where $0 < x < 1$ and $0 < \xi < 1$. (With the partial H-function used here it is necessary that neither x nor ξ should be equal to zero or unity. This limitation does not arise with the H-function itself, as will be seen when this function is defined.)

Find the H-differential coefficient at the point $x = \xi$ as follows:

$$\begin{aligned} f(\xi + \Delta x_1) &= [1 - (\xi + \Delta x_1)]\xi \\ f(\xi - \Delta x_2) &= [1 - \xi][\xi - \Delta x_2] \end{aligned} \tag{5}$$

$$R = \frac{[1 - (\xi + \Delta x_1)]\xi - [1 - \xi][\xi - \Delta x_2]}{\Delta x_1 + \Delta x_2}$$

$$= \frac{\Delta x_2 (1 - \xi) - \xi \Delta x_1}{\Delta x_1 + \Delta x_2} = \frac{\Delta x_2}{\Delta x_1 + \Delta x_2} - \xi \tag{6}$$

If $\frac{\Delta x_1}{\Delta x_2} = p$, as Δx_1 and Δx_2 tend to zero, the fraction $\frac{\Delta x_2}{\Delta x_1 + \Delta x_2}$ tends to the definite value $\frac{1}{1+p}$. Thus writing

$$\operatorname*{Lim}_{\Delta x_1,\ \Delta x_2 \to 0} R = \frac{Dy}{Dx} \tag{7}$$

where $\frac{D}{Dx}$ denotes H-differentiation to distinguish it from the ordinary derivative

$$\frac{Dy}{Dx} = \frac{1}{1+p} - \xi \qquad 0 \leq p \leq \infty \tag{8}$$

when $p = 0$

$$\frac{Dy}{Dx} = 1 - \xi$$

when $p = \infty$

$$\frac{Dy}{Dx} = -\xi$$

which coincide with the ordinary definitions of the derivatives $f'(x+0)$ and $f'(x-0)$. All the values of $\frac{Dy}{Dx}$ lying between $1 - \xi$ and $-\xi$ may be obtained by taking the different values of p from 0 to ∞.

The complete specification of the H-derivative of (4) is thus

$$\begin{aligned} y &= 1 - \xi & x &< \xi \\ y &= -\xi & x &> \xi \\ y &= \frac{1}{1+p} - \xi & x &= \xi \qquad (0 \leq p \leq \infty) \end{aligned} \tag{9}$$

and this in turn may be regarded as the definition of another H-function in the interval $0 < x < 1$. It will be noted that for $x < \xi$ and for $x > \xi$ it agrees with the derivative of the original function as obtained in the Theory of Functions of a Real Variable. At the point $x = \xi$, no differential coefficient exists according to the Theory of Functions of a Real Variable, as the derivative is discontinuous at this point. In the theory of

H-functions, however, as given here, the derivative at the point $x = \xi$ takes on every value between $1 - \xi$ and $-\xi$, the limits of the differential coefficients on the left and right obtained from both theories.

The partial H-function thus derived is an H-continuous function of a continuous variable. The meaning of H-continuity will be discussed later. The partial H-function has, however, an infinite sequence of values corresponding to the value $x = \xi$. It is now necessary to show how the definition given above of an H-derivative at a point at which the H-function has only one value can be extended to a point at which the H-function has an infinite number of values.

Applying the definition of an H-derivative, given above, to the formula

$$\begin{aligned} y &= 1 - \xi & x &< \xi \\ y &= -\xi & x &> \xi \end{aligned} \tag{10}$$

then

$$\begin{aligned} f(\xi + \Delta x_1) &= -\xi \\ f(\xi - \Delta x_2) &= 1 - \xi \end{aligned}$$

$$R = \frac{-\xi - 1 + \xi}{\Delta x_1 + \Delta x_2} = \frac{-1}{\Delta x_1 + \Delta x_2} \tag{11}$$

and

$$\frac{Dy}{Dx} = \lim_{\Delta x_1,\, \Delta x_2 \to 0} R = -\infty \tag{12}$$

If, now, in addition either Δx_1 or Δx_2 is allowed to be actually zero; take, for example, $\Delta x_1 = 0$, then

$$\begin{aligned} f(\xi) &= \frac{1}{1+p} - \xi \qquad 0 \leq p \leq \infty \\ f(\xi - \Delta x_2) &= 1 - \xi \end{aligned}$$

and

$$R = \frac{\frac{1}{1+p} - \xi - 1 + \xi}{\Delta x_2} = -\frac{p}{\Delta x_2 (1+p)} \quad (0 \leq p \leq \infty) \tag{13}$$

Hence

$$\frac{Dy}{Dx} = \lim_{\Delta x_2 \to 0} R = \lim_{\Delta x_2 \to 0} -\frac{p}{\Delta x_2 (1+p)} \quad (0 \leq p \leq \infty) \tag{14}$$

If $p = 0$

$$\frac{Dy}{Dx} = 0 \tag{15}$$

If $p = k\,\Delta x_2$

$$\frac{Dy}{Dx} = \operatorname*{Lim}_{\Delta x_2 \to 0} -\frac{k}{1 + k\,\Delta x_2} = -k \qquad (0 \leq k \leq \infty) \tag{16}$$

Similarly if Δx_2 is taken equal to zero, then at $x = \xi$

$$\frac{Dy}{Dx} = -l \qquad 0 \leq l \leq \infty \tag{17}$$

Thus the second H-derivative of the H_p-function

$$\begin{array}{ll} y = (1 - \xi)x & x \leq \xi \\ y = (1 - x)\xi & x \geq \xi \end{array} \tag{18}$$

is given by

$$\begin{array}{lll} y = 0 & & x < \xi \\ y = 0 & & x > \xi \\ y = -k & (0 \leq k \leq \infty) & x = \xi \end{array} \tag{19}$$

where $0 < x < 1$ and $0 < \xi < 1$, and this again may be regarded as an attempt to define another H_p-function. It is not a complete definition, however, and more data are required than that given in (19), as will be pointed out below in connection with the problem of H-integration of such an H_p-function. This H-derivative again agrees with that obtained in the Theory of Functions of a Real Variable when $x < \xi$ and $x > \xi$. It differs, however, at the point $x = \xi$, where it can take any value from 0 to $-\infty$.

The definition of H-differentiation can thus be extended to an H_p-function at a point where the H_p-function has more than one value as follows: Form the quantity

$$R = \frac{f(x_0 + \Delta x_1) - f(x_0 - \Delta x_2)}{\Delta x_1 + \Delta x_2} \tag{20}$$

in the same way as in the former definition. In this case, however, Δx_1 and Δx_2 may have any values including zero, but they cannot both be zero simultaneously. If Δx_1 or Δx_2 is zero, then $f(x_0 + \Delta x_1)$ or $f(x_0 - \Delta x_2)$ can have any of the values of the H_p-function at the point x_0. As before, fix a definite ratio of $\frac{\Delta x_1}{\Delta x_2} = p$. Then, keeping p fixed, the H-derivative of the H_p-function is given by

$$\frac{Dy}{Dx} = \operatorname*{Lim}_{\Delta x_1,\ \Delta x_2 \to 0} R \tag{21}$$

where the sign $\rightrightarrows 0$ is understood to mean that Δx_1 and Δx_2 may both approach zero, or Δx_1 may be zero and the limit taken as Δx_2 approaches zero or *vice versa,* but Δx_1 and Δx_2 cannot both be equal to zero simultaneously.

Again as before, if this limit is independent of p the H-derivative has only one value at the point, and this is equal to the limiting values of the H-derivatives on the right and on the left. If the limit is not independent of p, the H-derivative can have all the values of this limit as p varies from 0 to ∞.

If now the H-derivative of this latter H_p-function given by (19) is taken, the third H-derivative of (18) is found to be

$$\begin{array}{lll} y = 0 & & x < \xi \\ y = 0 & & x > \xi \\ y = -k & (-\infty \leq k \leq \infty) & x = \xi \end{array} \qquad (22)$$

and continuing this the fourth H-derivative is

$$\begin{array}{lll} y = 0 & & x < \xi \\ y = 0 & & x > \xi \\ y = -k & (-\infty \leq k \leq \infty) & x = \xi \end{array} \qquad (23)$$

From these last two statements, (22) and (23), it would appear as if the third and fourth H-derivatives of (18) were the same H_p-function. The difference between them will be discussed in connection with the definition of an H-integral. It suffices to say here that the above statements are not definitions, but only attempts to define the H_p-functions, and it is only due to the insufficiency of the definition that they appear the same.

H-INTEGRATION.

The problem presented in integrating H_p-functions, such as those derived above by H-differentiation, will now be considered. For points at which the H_p-function to be integrated remains finite, the ordinary theory of integration might possibly suffice, but it would be useless at a point where the H_p-function took a series of values from 0 to ∞, although the H_p-function from which it was derived by H-differentiation remained finite.

The only definition which was found to be of value in such a

case was to define H-integration as the inverse process of H-differentiation. When thus defined, if

$$\frac{Dy}{Dx} = Q \tag{24}$$

then

$$\#\, Qdx = y + C \tag{25}$$

where $\#$ denotes H-integration, and C is the usual arbitrary constant of integration. This equation holds exactly, provided y is an H_p-function. It immediately follows from this definition that if an H_p-function is H differentiated n times, and then H integrated n times, no matter how many infinities arise in the process, the original H-function is obtained, provided the constants of integration are properly chosen. To the question as to how we are going to deal with infinities if they arise, we can only answer that the very definition of the H-integral says that if the n^{th} H-derivative is H integrated n times the original H_p-function is obtained. Another question might be asked, "How is it known there is an inverse process to H-differentiation?" It appears that the only answer that can be given to such a question is that we either know intuitively or make it one of our postulates that to every process in the theory of H-functions there is an inverse. Of course such intuitive knowledge may be objected to, but it is always necessary to have either some facts or some assumptions to start with, if a start is to be made at all. This will be taken as one of our facts. It will be found later in discussing H-limits and H-continuity that these concepts give reasons for the further assumption of intuitive facts.

With this definition it now becomes perfectly easy to integrate the first H-derivative given in (9), *i.e.*,

$$\begin{aligned} y &= 1 - \xi & x &< \xi \\ y &= -\xi & x &> \xi \\ y &= \frac{1}{1+p} - \xi & x &= \xi \end{aligned} \tag{26}$$

and get the H_p-function

$$\begin{aligned} y &= (1-\xi)x + C & x &\leq \xi \\ y &= (1-x)\xi + C & x &\geq \xi \end{aligned} \tag{27}$$

The proof that (27) is the H-integral of (26) is simply that when the H-derivative of (27) is taken (26) is obtained. The third equation in (26) enables us to say that the constant C in the first equation of (27) when $x < \xi$ is the same as the constant C in the second equation of (27) when $x > \xi$.

When the second H-derivative given in (19)

$$\begin{array}{lll} y = 0 & & x < \xi \\ y = 0 & & x > \xi \\ y = -k & (0 \leq k \leq \infty) & x = \xi \end{array} \tag{28}$$

is H integrated twice, (26) is obtained, and again the proof is that by taking the second derivative of (26), (28) is obtained. But now we have to examine (28) more closely and consider what would happen if equations (28) alone were given and it was required to find the H-integral. It could not be done, and this is due to the insufficient data given in (28). There are two things we would have to know. First, is the integral finite at the point $x = \xi$; and second, if so, what is its value at this point? Both these pieces of information might perhaps be given by writing the last equation in (28) in the form

$$y = -k \qquad (0 \leq k \leq 1\,\infty^1) \qquad x = \xi \tag{29}$$

where the symbol $1\,\infty^1$ is to be interpreted as follows: The symbol ∞^1 denotes that it is what we may call an integrable infinity of the first order, by which is meant that the first H-integral has a finite value, and the symbol 1 in front signifies that the maximum value of the integral at the point $x = \xi$ is unity.

Thus a specification of the second H-derivative of the H_p-function

$$\begin{array}{ll} y = (1 - \xi)x & x \leq \xi \\ y = (1 - x)\xi & x \geq \xi \end{array} \tag{30}$$

is given by

$$\begin{array}{lll} y = 0 & & x > \xi \\ y = 0 & & x < \xi \\ y = -k & (0 \leq k \leq 1\,\infty^1) & x = \xi \end{array} \tag{31}$$

If the original H_p-function had been

$$\begin{array}{ll} y = (2 - \xi)x & x < \xi \\ y = (2 - x)\xi & x > \xi \end{array} \tag{32}$$

it can easily be verified that the first H-derivative is

$$\begin{aligned} y &= 2 - \xi & x &< \xi \\ y &= -\xi & x &> \xi \\ y &= \frac{2}{1+p} - \xi & x &= \xi \end{aligned} \tag{33}$$

and then in accordance with the definition of the symbol $2\infty^1$ above, the second H-derivative is

$$\begin{aligned} y &= 0 & x &< \xi \\ y &= 0 & x &> \xi \\ y &= -k \quad (0 \leq k \leq 2\infty^1) & x &= \xi \end{aligned} \tag{34}$$

and similarly for any other such H_p-functions.

Again if the third H-derivative already obtained in (23) were given, it would be impossible to H-integrate this H_p-function as there defined without further data. This might perhaps be given as follows: The positive infinity is an H-integrable infinity of the second order, by which is meant that when one H-integration is applied to it an integrable infinity of the first order is obtained. The symbol $1\infty^2$ will be used to denote the H-integrable infinity of the second order, and thus a specification of the third H-derivative is given by

$$\begin{aligned} y &= 0 & x &< \xi \\ y &= 0 & x &> \xi \\ y &= -k \quad (-(0)\infty^1 \leq k \leq 1\infty^2) & x &= \xi \end{aligned} \tag{35}$$

when the $-(0)\infty^1$ denotes the lower value of the H-integral is an H-integrable infinity of the first order whose maximum value on H-integration is zero.

Similarly the fourth H-derivative may be given by

$$\begin{aligned} y &= 0 & x &< \xi \\ y &= 0 & x &> \xi \\ y &= -k \quad (-(0)\infty^2 \leq k \leq 1\infty^3) & x &= \xi \end{aligned} \tag{36}$$

It will be admitted that these symbols $1\infty^3$, etc., are perhaps not the best way to proceed in defining these various H-derivatives. They have, however, naturally suggested themselves in connection with the problem of H-integration. Subsequent work, however, seems to point to the fact that in order to define these various derivatives it is best to give the original H_p-function in (30) and then call them the first, second, third, etc., H-derivatives of this H_p-function, and if the numerical values of the derivatives

are required they can be obtained from (26), (31), (35), etc. The notation $1\,\infty^3$, etc., however, has the advantage of simplicity and may be used until the necessity of this latter procedure is found.

The objection may be raised, suppose that a function is defined by

$$\begin{array}{llll} y = 0 & & x < \xi & \\ y = 0 & & x > \xi & (37) \\ y = -k & (0 \leq k \leq \infty) & x = \xi & \end{array}$$

How should we proceed to find the H-integral? The answer should be plain from the above. Equation (37) is not the complete definition of an H_P-function, due to the fact that enough information is not given in the third equation. Since it is not an H_P-function, the operation of H-integration cannot be applied to it and hence there is no problem.

It may, of course, be asked where Riemann's definition, Lebesgue's definition, etc., of integrals fit in this scheme. All of these definitions comprise modes for dividing up the interval into subdivisions and finding the value of the integral in one of these modes of subdivision. The limit of the sum of a number of infinitesimal integrals is taken as the subdivisions become indefinitely small. To discuss this summation completely will require a discussion of infinite series and must be deferred until then. It may, however, be said at present that these definitions of an integral as a sum may all be regarded in the theory of H-functions as attempts to approximate to the conception of H-integration being the inverse process of H-differentiation. They may be used for such a purpose as a general rule, though a little care is needed in their use, such as at points of non-uniform convergence, etc. This will be taken up after a discussion of infinite series.

H-SUMMATION OF SERIES.

The subject of summation of series may be introduced by referring to an interesting discussion which will be found in *Nature*, Vols. 58, 59, 60, in letters to the Editor under the title of "Fourier Series," in which Michelson, Gibbs, Baker, Love and others took part. We shall not take the particular series they discussed there, but will take, for reasons which will be apparent later, the so-called Fourier expansion for the function used in the

above examples on H-differentiation and H-integration defined in (4). The Fourier series is

$$y = \frac{2}{\pi^2}\left[\sin \pi x \sin \pi\xi + \frac{\sin 3\pi x \sin 3\pi\xi}{3^2} + \frac{\sin 5\pi x \sin 5\pi\xi}{5^2} + \ldots\ldots\right] \quad (38)$$

The method to be pursued in summing this series in accordance with the rules laid down in the Theory of Functions of a Real Variable (they will be found in Carslaw, "Fourier Series and Integrals," or other works on the subject) is as follows:

When we speak of the sum of an infinite series

$$u_1(x) + u_2(x) + u_3(x) + \ldots\ldots \quad (39)$$

it is to be understood:

(1) That we settle upon the value of x for which we wish to sum the series;

(2) That we insert this value of x in the different terms of the series;

(3) That we find the sum—$S_n(x)$—of the first n terms;

(4) That we find the limit of this sum as $n \to \infty$, keeping x all the time at the value settled upon.

This rule for summing a series apparently follows directly from the definition of a function as follows: Let a and b be any two real numbers $b > a$. If to every value of x in the interval $a \leq x \leq b$ there corresponds a real number y, then we say that y is a function of x and we write $y = f(x)$.

Now it will be noted that in this definition of a function no mention is made as to whether the variable x is continuous, and in finding the sum of the series this is not assumed. If reference is made to the definition already given of an H_P-function, it will be seen that one of the conditions was that the variable x is continuous. It also required that y be a continuous function in this interval. We shall now endeavor to find a rule for summing series such as (38) and its derivatives, which will allow the sum to be continuous and thus permit it to be regarded as an H-function.

The series (38) is uniformly convergent at every point in its range, as may be shown by the Theory of Functions of a Real Variable. Differentiate it with respect to x, then

$$y' = \frac{2}{\pi}\left[\cos \pi x \sin \pi\xi + \frac{\cos 3\pi x \sin 3\pi\xi}{3} + \frac{\cos 5\pi x \sin 5\pi\xi}{5} + \text{etc.}\right] \quad (40)$$

changing the origin from $x = 0$ to $x = \frac{1}{2}$, we get

$$y_1' = \frac{2}{\pi}\left[\sin \pi x \cos \pi\xi + \frac{\sin 3\pi x \cos 3\pi\xi}{3} + \frac{\sin 5\pi x \cos 5\pi\xi}{5} + \text{etc.}\right] \quad (41)$$

and putting $\xi = 0$ in this equation

$$y_2' = \frac{2}{\pi}\left[\sin \pi x + \frac{\sin 3\pi x}{3} + \frac{\sin 5\pi x}{5} + \ldots\ldots\right] \quad (42)$$

which is very similar to the series for which Carslaw discusses Gibbs' phenomenon and is also very like the series Michelson discusses in his letters to *Nature*. When x is not equal to zero or π, the sum of the series can be obtained from the Theory of Functions and is equal to $\frac{1}{2}$, and this value is obtained no matter how small x may be taken, as long as it is not actually zero. When $x = 0$, all the terms are zero, the limit which $S_n(x)$ approaches is zero, and this produces the well-known discontinuity in the sum obtained in the Theory of Functions. No use has been made yet of the fact that if (38) is regarded as an H-function, the variable x is continuous, as it is also in (42). In order to make use of this fact let $x = 1/p$, where p is a very large number. Then the series can be written in the integral form (Note: This is the ordinary integral, not the H-integral, which may be used in the case). Then

$$y_2' = \frac{1}{\pi}\sum_{n=1}^{n=\infty} \frac{\sin \frac{2n-1}{p}}{\frac{2n-1}{p}} \cdot \frac{2}{p} \longleftrightarrow \frac{1}{\pi}\int_0^{\frac{2n}{p}} \frac{\sin \alpha}{\alpha}\, d\alpha \quad (43)$$

(The use of the sign $\longleftrightarrow$, which may be read "converges to," will be discussed later.)

The only restriction on n and p is that they are both positive numbers, and both approach the limit ∞. The ratio of n to p is perfectly arbitrary and may be put equal to k, so that

$$y_2' \longleftrightarrow \frac{1}{\pi}\int_0^k \frac{\sin \alpha}{\alpha}\, d\alpha \qquad 0 \leq k \leq \infty \quad (44)$$

For the values of this integral, reference may be made to a paper by Bôcher.[4] The maximum value occurs when $k = \frac{\pi}{2}$, giving Gibbs' phenomenon for the series which is that the maximum difference between the various values at $x = \xi$ in the original

[4] *Amer. Math. Jour.*, Princeton, N. J., Series 2, 7, p. 124.

series approaches the value $\frac{2}{\pi}\int_0^{\pi}\frac{\sin\alpha}{\alpha}\,d\alpha$ or $\frac{1.8519}{1.5708}$ approx.; while the maximum difference between the right and left in the sum of the series is only 1 at $x=\xi$. Gibbs' phenomenon in the Theory of Functions applies only to the approximation curves, but in the form stated above it may be called the Gibbs' phenomenon in the theory of H-functions.

The H-sum of this series may thus be conceived of as the limit of the sums obtained by summing the series to n terms. Curves showing the forms obtained will be found in Carslaw or other works on Fourier series.

The method given here for summing the series was suggested by Michelson in his letters to *Nature*, but was objected to by Baker, Love and others on the grounds that it did not follow the rule which they gave (see above), since when we let $x=1/p$ and then let $k=n/p$, where k may vary, we have not named our value of x as required in paragraph (1) of the rule. This objection cannot arise in taking the H-sum of the series as an H-function, because the hypothesis is that x is continuous, and the only way we can make x approach zero in a continuous manner is to let $x=1/p$ and then let p become larger and larger. We are still in the difficulty, however, that if we name a value of p, no matter how large, it fixes x at a small, though definite, interval from zero. But we find that we have also at our disposal another quantity n which has to become indefinitely large, and that by a little ingenuity we can arrange it so that we get n and p, in the form of a fraction, n/p. What could be more natural than to assume that, since n and p are perfectly arbitrary and very large, we may let this ratio equal to k where k is an arbitrary number. We thus avoid naming a definite value of x for which we are going to sum the series before we name the value of n, the number of terms to which we are going to sum the series.

(To be continued.)

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

THE USE OF COLOR FOR THE EMBELLISHMENT OF THE MOTION PICTURE PROGRAM.[1]

By L. M. Townsend and L. A. Jones.

A DESCRIPTION is given of various types of spot and flood-light units used in the Eastman Theatre for the production of the color effects used as accessories to the motion picture program. In the introductory part of the paper some space is devoted to a review of various attempts which have been made to employ color in the abstract form as a means of entertainment and emotional appeal, and a résumé of color terminology and knowledge is given. Laws of color mixture, both additive and subtractive, are stated briefly. A duplex projecting system using pattern plates complementary both in form and tone, in conjunction with compound color filters for the production of dynamic color effects, is described. The compound color filters are made up by using each of the three-color subtractive primaries in the form of wedge filters. An improved form of stereopticon spot and flood-light units is described and their use in the production of various types of static and dynamic color effects discussed in detail.

PHOTOGRAPHIC GELATIN.[2]

By S. E. Sheppard.

PHOTOGRAPHIC gelatins contain a sensitizing substance or substances which increase the speed of silver bromide emulsions, and R. F. Punnett has obtained sensitizing extracts from active gelatin. This photographic sensitizer has now been extracted from by-product liquors in gelatin manufacture, seeds and other vegetable materials, and identified as a mustard oil, *i.e.*, an organic,

* Communicated by the Director.

[1] Communication No. 237 from the Research Laboratories of the Eastman Kodak Company and published in *Trans. Soc. Mot. Pic. Eng.*, No. 21, 38 (1925).

[2] Communication No. 240 from the Research Laboratories of the Eastman Kodak Company, and published in *Phot. J.*, 65, 380 (1925).

generally *allyl isothiocyanate*. This substance in the presence of ammonia is converted to *allyl thiocarbamide* (thiosinamine), which reacts with silver halides forming double silver compounds which decompose to give *silver sulphide*. The hypothetical sensitivity centres of silver bromide grains have an actual existence and are composed of silver sulphide. It is shown that this sensitizing is a general property of substances combining a doubly bonded sulphur, selenium or tellurium atom, and that inorganic thiosulphates (iso), selenocyanates, etc., behave in the same way. The formation of intermediate double compounds with silver halide is an important phase. The amounts of sensitizer in photographic gelatins are very small, being of the order of 1 in 300,000 parts of gelatin.

FOG CORRECTIONS IN PHOTOGRAPHIC DENSITIES.[2]

By R. B. Wilsey.

IN SENSITOMETRY it is necessary to correct observed densities for the silver reduced without exposure, which is generally known as "fog." This is usually done by measuring the density on an unexposed area and deducting the fog reading so obtained uniformly from all observed densities. More accurately, the rate of growth of fog over the image should be proportional to the mass of unaffected silver halide. Meidinger assumes that the fog over the image is proportional to the number of grains of silver halide which have not developed as exposed grains. This method is criticized, and a modification proposed, in which fog over the image is assumed to be proportional to the mass of silver unaffected by exposure. This gives rise to the equation, $D_f = (D_m - D_\infty)/D_m.F$, where D_f is the fog correction to be deducted, D_m the maximum density developable on the plate, D_∞ the limiting image density for the given exposure, and F the fog density observed on unexposed areas of the plate. A method of obtaining an approximate $D_\infty - \text{Log } E$ curve for purposes of fog correction is described. This fog correction should be adequate for all ordinary sensitometric data, where fog is not excessive and where there are no special or unusual disturbing factors.

[2] Communication No. 246 from the Research Laboratories of the Eastman Kodak Company, and published in *Phot. J.*, **65**, 454, 1925.

CHEMISTRY OF THE ACID-FIXING AND HARDENING BATH.

PART II.[4] THE PRECIPITATE FROM THE INTERACTION OF DEVELOPER WITH ACID-FIXING AND HARDENING BATHS.[5]

By S. E. Sheppard and A. Ballard.

IT HAS been suggested that a definite basic aluminum sulphite is precipitated by the interaction of developing solutions (containing sulphite of soda) and acid-fixing baths containing alum. Our investigation has shown that the composition of this precipitate is not constant, but is a function of the alkalinity of the solutions. The more alkaline the final solution, the lower the SO_2 content of the precipitate; ultimately the precipitate consists entirely of hydrous alumina.

EXPERIMENTAL.

A series of precipitates were made by adding 25 c.c. of a solution containing .6672 gm. $AlCl_3$, equivalent to .1355 gm. Al to 25 c.c. of a solution containing invariably .944 gm. Na_2SO_3 and a variable amount of Na_2CO_3. The precipitate was dried on a filter and washed with ten (10) changes of water, and titrated with iodine and thiosulphate.

No.	Gm. Na_2CO_3.	pH.	C.c. N/10 I_2.	Gm. SO_2.
1	0	3.8	16.0	.0512
2	.25	6.6	12.35	.0384
3	.50	7.6	5.4	.0173
4	.75	8.4	1.25	.0040

Supposing a hydrolyzable basic sulphite present in the precipitate, some error might be produced by the washing, but this would be approximately constant. The data point rather to a mixture of normal aluminum sulphite and hydrous alumina, the proportion of the latter increasing with the alkalinity.

Similar results were obtained with a mixture of alum, sodium sulphite and acetic acid, which was precipitated with different amounts of sodium carbonate. The solution contained 3.00 gm. alum, 1.50 gm. sodium sulphite (Na_2SO_3) and 2.5 c.c. acetic acid (glacial). The precipitate was decanted and repeatedly washed with distilled water.

[4] Communication No. 249 from the Research Laboratory of the Eastman Kodak Company.

[5] *Cf.* S. E. SHEPPARD, F. A. ELLIOTT and S. S. SWEET: JOUR. FRANK. INST., July, 1923, p. 45.

No.	Gm. Na_2CO_3.	pH.	C.c. N/10 I.	SO_3/Al_2O_3.
1	1.			
2	1.2	4.6	4.38	28.6 : 100
3	1.8	5.6	4.48	24.6 : 100
4	2.5	6.5	3.40	19.6 : 100

Two sets of experiments were made in which actual developing solution, containing sulphite and carbonate, was added to an acid-fixing and hardening bath. In the first series, there was insufficient added to precipitate all of the alumina, in the second more than sufficient:

Series I: Ratio of SO_2 to Al_2O_3, 20 : 100.

Series II: No SO_2 found in precipitate.

Further, when alum was added to 30 per cent. "Hypo" solution, the precipitate formed was identified as sulphur. No aluminum thiosulphate appeared to be formed.

CONCLUSION.

The ratio of sulphurous acid to alumina in the precipitates formed when alkaline sulphite and developing solutions are added to an acid-fixing bath depends upon the final acidity (or alkalinity) of the solution in equilibrium. The sulphurous acid content decreases steadily as the alkalinity increases. It is possible that a series of basic aluminum sulphites, adsorbed to hydrous alumina, are formed, but no direct evidence of their formation was obtained, the composition of the precipitate varying continuously.

Motorship Tonnage. (*Compressed Air Magazine*, Sept., 1925, p. 1386.)—Although the statement issued by Lloyd's Register of Shipping for the first three months of the current year shows shipbuilding generally to be at its lowest point since the World War, nevertheless the decline has one outstanding significant feature. That is to say, while the building of steam tonnage has fallen steadily for some time, the orders for the construction of vessels to be driven with oil engines have multiplied to a marked degree. This trend has been so pronounced that nearly half the world's shipbuilding is of this newer type. An especially interesting angle of this movement was disclosed not long ago when the fact was brought to light that ship owners rather than shipbuilders and engineers were responsible for the demand for oil engines instead of steam engines for prime movers. In short, the practical man—the owner and operator—the man that has to make his ship pay, has chosen the motor-driven vessel because of its proved economies. R.

NOTES FROM THE U. S. BUREAU OF CHEMISTRY.*

SALTBUSHES AND THEIR ALLIES IN THE UNITED STATES.[1]

By G. L. Bidwell and E. O. Wooten.

COVERING many millions of acres of the arid and semi-arid western states are various native plants, upon which millions of meat animals graze each year. As comparatively little seemed to be known about the forage value of these plants, the Department of Agriculture (Bureau of Plant Industry and Bureau of Chemistry) began a study of them several years ago. The information on certain of the native grasses was published in 1915 as Department of Agriculture Bulletin 201. The information on some of the saltbushes and their allies appears in Department of Agriculture Bulletin 1345.

This bulletin gives the botanical characteristics, the chemical composition, and the food value of many members of the saltbush and allied families. A list of their common names, with the scientific equivalents, is appended. The non-technical description, supplemented by the illustrations, will help stockmen on the dry lands of the West to identify the different plants of these botanical families.

COMMERCIAL DEHYDRATION OF FRUITS AND VEGETABLES.[2]

By P. F. Nichols, C. R. Gross, and W. A. Noel.

THE outstanding advantage of drying as a method of preserving foods is that the weight and bulk of the products are greatly reduced, thus making possible economy in storage and transportation. This saving is of special significance during periods when the need for food conservation is greatest. Such a need existed during the World War and it was at that time that the Bureau of Chemistry undertook an extensive investigation of the commercial dehydration of fruits and vegetables. The results

* Communicated by the Chief of the Bureau.

[1] Department of Agriculture Bulletin 1345. Issued September, 1925 (contribution from Bureau of Chemistry and Bureau of Plant Industry).

[2] Department of Agriculture Bulletin 1335. Issued September, 1925.

of this study are incorporated in Department of Agriculture Bulletin 1335.

This bulletin points out the advantages and disadvantages of the preservation of fruits and vegetables by dehydration and shows the extent of the industry. It gives details for the construction and equipment of a commercial dehydration plant and explicit directions for the entire process, from the selection of the fresh material to the packing of the finished product. Engineering calculations to aid in designing a drier and preventive and remedial measures against insects which attack dried fruits are included.

A COLORIMETRIC METHOD FOR SHOWING THE DISTRIBUTION AND QUANTITY OF LEAD ARSENATE UPON SPRAYED AND DUSTED SURFACES.[3]

By C. C. Hamilton and C. M. Smith.

THE usual methods of recording the distribution and determining the quantity of lead arsenate upon sprayed surfaces having proved to be inadequate in a study of spreaders conducted in the University of Maryland, shorter and easier methods were developed in conjunction with the Bureau of Chemistry. Several methods for recording the distribution of lead arsenate were tried, such as ordinary photographs, X-ray photographs, and microscopical examinations, but none proved as satisfactory as the "printing out" chemical method. The method for determining the quantity of lead arsenate was developed from this. Both depend upon transposition of the lead arsenate to lead sulphide. Repeated tests have shown that, with careful manipulations, these methods are sufficiently accurate. The probable error is less than variations due to differences in spraying. The manipulations are easily and rapidly made and do not require much or costly apparatus.

BACTERIAL CONTENT OF HEAD LETTUCE.[4]

By Lawrence H. James.

FRESH lettuce was found to carry 10,000,000 organisms per gram on the outer leaves and 320,000 per gram on the centre leaves. Holding at icebox temperature allowed marked multi-

[3] Published in *J. Econ. Entomol.*, **18** (June, 1925), p. 502.

[4] Published in *Amer. Food J.*, **20**, No. 6 (June, 1925), p. 302.

plication of organisms, reaching 20,000,000 on the inside leaves. These are non-pathogenic, putrefying organisms.

Washing head lettuce before storing in the icebox causes much more decay than storage without washing. The bacterial count on unwashed heads stored for seven days was 10,000,000, while on washed heads it was 2,000,000,000 bacteria per gram. Lettuce would keep longer and better if placed in the icebox as soon as bought, being thoroughly washed just before using.

SPECTROPHOTOMETRIC DETERMINATION OF HYDROGEN-ION CONCENTRATIONS AND OF THE APPARENT DISSOCIATION CONSTANTS OF INDICATORS.

IV. 1-NAPHTHOL-2-SODIUM SULPHONATE INDOPHENOL.[5]

By Walter C. Holmes and Edward F. Snyder.

A STUDY in the Bureau of Chemistry and Bureau of Plant Industry has shown that 1-naphthol-2-sodium sulphonate indophenol is relatively unstable in aqueous solutions, but is stable in absolute alcohol, that the dissociation of the dye follows the normal course of a monobasic acid, and that the apparent dissociation constant is 8.63 in aqueous solutions containing 5 per cent. of alcohol. Data have been obtained for the application of the indicator in the determination of hydrogen-ion exponents by the spectrophotometric and drop-ratio methods.

CONSTITUENTS OF CRUDE COTTONSEED OIL.[6]

By G. S. Jamieson and W. F. Baughman.

FURTHER progress has been made in the investigation assigned to the Oil, Fat and Wax Laboratory at the conference of the Basic Research Committee and the Department of Agriculture.

A small quantity of so-called vegetable mucilage was separated from crude cottonseed oil. It was precipitated from an aqueous solution by normal lead acetate. No water-soluble gum could be detected. Experiments showed that this mucilage readily emulsifies the oil with water and that the emulsion does not break after standing for several days. It is believed that the mucilage constitutes only a very small part of the non-glyceride substances present in the oil.

[5] Published in *J. Am. Chem. Soc.*, **47**, No. 8 (August, 1925), pp. 2232–2236.

[6] Published in *Jour. Oil and Fat Ind.*, **2**, No. 3 (July, 1925), pp. 101–105.

Treating the oil with water and then extracting the separated oil repeatedly with large volumes of alcohol removes only part of the resin. The resin is partially removed from an alcoholic solution by an alcoholic solution of copper acetate in the form of a dark yellow flocculent precipitate. The copper salt is insoluble in alcohol and ether. It is believed that part of the deep red color of the crude oil is due to this resin.

Magnetization and Magnetic Hysteresis in Permalloy. O. E. BUCKLEY and L. K. MCKEEHAN. (*Phys. Rev.*, Aug., 1925.)—To get material of exceptionally pure content, specimens of iron and nickel were chemically purified, powdered, sintered in vacuum, swaged and from the resulting mass there were drawn wires about 60 cm. long and .1 cm. in diameter. Permalloy of 81 per cent. nickel and 19 per cent. iron was scarcely altered in its magnetic behavior by the application of a tension of 10,000 lb. per sq. in., while other specimens containing 84 and 65 per cent. nickel were changed by tension. The alloy with less than 81 per cent. nickel is more easily magnetized and has less hysteresis when under tension than when unstretched, while specimens with higher nickel content show just the opposite relations. Permalloy with 78.5 per cent. nickel under a tension of 3560 lb. per sq. in. is saturated practically by a field of only .2 gauss. Then, when cyclically magnetized, its hysteresis loss per c.c. per cycle amounts to but 80 ergs. Soft iron with an induction of 15,000 has a hysteresis loss of 6500, or eighty times as much as for the permalloy with its induction of 11,000. For K. S. magnet steel with induction of 16,000, the loss is as much as 900,000 ergs per c.c. per cycle. The smallness of the hysteresis loss in permalloy leads the authors to say, "Since it is unreasonable to suppose that any of the samples tested were perfectly homogeneous, it may well be that in small regions hysteresis completely disappeared in certain of the experiments." In the case of the 78.5 per cent. nickel permalloy the permeability is much greater than that of soft iron. For instance, when the induction is 6000 for both magnetic substances, the permeability of the permalloy is about six times as much as for the iron. When a tension of 3560 lb. per sq. in. is applied to the permalloy wire the ratio of permeabilities rises to about 28.

The application of weak magnetic fields causes iron to lengthen, while the length of nickel is shortened by the application of a magnetic field of any strength. These effects are named "magnetostriction." It is found that high-nickel permalloy contracts like nickel and that permalloy low in nickel lengthens like iron. Somewhere between 80 and 83 per cent. nickel content the magnetostriction changes its sign. G. F. S.

NOTES FROM THE U. S. BUREAU OF MINES, DEPARTMENT OF COMMERCE.*

BARIUM POLYSULPHIDE IN SULPHIDIZING OXIDIZED ORES FOR FLOTATION.

By Edmund S. Leaver and Horace M. Lawrence.

For the investigation two fairly typical oxidized ores of copper and lead were selected. The copper ore came from south-eastern Utah, and assayed 3.8 per cent. Cu; the copper was present in a sandstone gangue, mainly as the carbonates, malachite and azurite, with less than 0.2 per cent. as sulphides, or silicate. The lead ore is from the Tintic district, Utah. It is typical of the oxidized lead-silver ores of that district; the lead is present in an oxidized form in a siliceous gangue, much discolored by ferric oxide. The sample assayed 5.15 per cent. Pb and 14.3 oz. Ag.

In addition there were available some results secured by Mr. J. B. Lain, consulting metallurgist of Reno, Nev., on a sample of ore from the Chino Copper Company. The ore assayed about 1.7 per cent. Cu, of which about 0.25 per cent. was non-sulphide.

The sulphidizing reagents used were hydrogen sulphide, ammonium polysulphide and sulphides and polysulphides of the alkaline and alkaline-earth metals.

Both ores were crushed to 10 mesh and carefully sampled. As preliminary experiments had shown that fine grinding was required, the ore for all tests was ground to pass a 65-mesh screen. For frothing and collecting purposes, the reagents used were coal-tar, coal-tar and hardwood creosotes, ortho-toluidine and thiocarbanilide (T and T mixture), and pine oil, in amounts of 1.0 to 3.0 lb. per ton of ore, depending upon the reagents used.

The results show that barium polysulphide has possibilities as a sulphidizing reagent. On the copper ore the extraction and grade of concentrates were about as good as were obtained from any of the sulphidizing reagents used. On the lead ore, however, barium polysulphide did not give as high an extraction as was secured with a combination of sodium sulphide and polysulphide, or the sodium sulphide-paraffin mixture. Barium polysulphide prepared by either of the two methods seemed to have about the

* Communicated by the Director.

same sulphidizing effect. In districts where deposits of barite occur, barium polysulphide might be prepared cheaply, and prove to be a satisfactory sulphidizing reagent.

Of the other sulphidizing reagents considered in this investigation sodium sulphide and sodium polysulphide, either separately or as a combination, gave the best results. Calcium polysulphide used alone did not yield as good results as had been expected, as it has been somewhat favored in the past by investigators, partly because of its comparative cheapness. Sodium sulphide and calcium polysulphide as a combination proved to be satisfactory for the copper ore, and at least one of the companies treating oxidized copper ores uses this combination successfully. Potassium and ammonium polysulphides are too expensive for commercial practice, particularly as at least equal or larger amounts of these reagents are needed than is required with the cheaper sulphidizing reagents. In no case in the amounts used did hydrogen sulphide prove to be as satisfactory a sulphidizing reagent as some of the others. Additional information will be found in serial 2698, recently published.

THE METALLURGY OF QUICKSILVER.

By L. H. Duschak and C. N. Schuette.

IN THE years 1850 to 1923, the United States produced 2,426,000 flasks[1] (73,600 metric tons) of quicksilver worth $120,500,000. California yielded 2,195,000 flasks of this total; the remainder came from Texas, Oregon, Nevada, and Arizona. Most of this metal has been extracted from low-grade ores, those containing less than 0.5 per cent. mercury or 10 pounds per ton. The finished product or "virgin metal" is made at the mine and shipped in flasks.

Quicksilver is unique in being the only metal that is liquid at ordinary temperature. Because of this and other physical and chemical properties, it is perhaps more indispensable to industry than any other metal. On the other hand, the quicksilver industry of the world is of vanishing significance[2] when compared with

[1] Statistics from U. S. Geological Survey. One flask now contains 75 lb.; formerly up to 1904, 76½ lb.

[2] If the new mercury boiler should prove as successful in practice as its sponsors believe, it will cause an increased demand for quicksilver, but this demand is not likely to be great in the immediate future: Quicksilver production for 1923, U. S. Geol. Survey, June 23, 1924.

the major mineral industries with respect to quantity and value of product, capital invested, or the number of men employed. The peculiar value of quicksilver is due to the fact that in some of its applications no substitute is available and in others the substitutes would be unsatisfactory or extremely expensive. Scarcely a branch of science or industry fails to make some use of mercury or its compounds. As a detonator for explosives, mercury fulminate holds first place and in safety and reliability could be replaced only by the highly expensive silver fulminate.[3] Through its use in detonators and in the metallurgy of the precious metals, quicksilver is of special importance to mining. In medicine, in the manufacture of electrical apparatus, in the production of pigments and antifouling paints, and in the general field of experimental science, quicksilver is equally indispensable.

Because of its small commercial importance and the lack of a stable market and price for the metal, the quicksilver industry, as a whole, has not had the benefit of the same metallurgical and business direction that has been given to the winning of the major metals. The unique relation of mercury to national health (certain drugs) and security (fulminate for defensive purposes), and the indispensability of the metal and its compounds in science and industry seem to justify investigation of the quicksilver industry by the Bureau of Mines. In Europe, government interest has been direct. The rich deposits at Almaden, Spain, are owned and the product is marketed by the Spanish Government; the mines at Idria, formerly belonging to the Austro-Hungarian Government, have now passed under Italian control; and the most productive mines of the Monte Amiata district, Italy, which were largely owned by German interests, were taken over by the Italian Government after Italy's entry into the World War. While under the control of the Austro-Hungarian Government, the mines and reduction works at Idria employed a number of able engineers, and, as a result, notable advances in the metallurgy of quicksilver were made there.

In the United States the quicksilver deposits which can supply domestic needs for many years to come are of course privately owned; and the industry has so far received a little, if any, assis-

[3] According to S. P. Howell and J. E. Crawshaw, explosives engineers of the Bureau of Mines.

tance from the Federal Government, except in the matter of tariff protection.[4] An opportunity to be of assistance was offered the Bureau of Mines in 1917, to study certain quicksilver condensers, with particular reference to eliminating sources of loss that might exist in the treatment of low-grade ores. The matter was investigated by the bureau jointly with several operating companies in California and the results have been published.[5]

The writers have recently completed a general study of the industry. The main conclusions to be drawn from the work may be summarized as follows:

In the direct-furnace treatment of quicksilver ores the major problem in the extraction of quicksilver has been solved. Methods are available whereby low-grade ore can be treated with a remarkably high recovery and at low cost in view of the small scale of operations at most plants. It does not follow that present practice at all, or even at the majority of the quicksilver reduction works in this country, has reached the highest possible point of efficiency. Improvements can be made at many plants mainly by correcting minor defects rather than by making fundamental changes in the process used.

In the past the quicksilver industry has suffered from lack of competent technical supervision, and some time and effort have been wasted through attempts to devise improvements in process and equipment without adequate regard for developments in other branches of metallurgy. Now knowledge of the metallurgy of quicksilver has advanced so far that adequate information is available for the design, construction, and operation of a plant for the treatment of any ordinary mercury-bearing ore. A few unusual cases require further investigation, but generally speaking there is no difficulty in recovering the metal from its ores effectively, and improvements in practice will consist mainly in applying available information more efficiently.

Mining of the ore offers the greatest opportunity for reducing production costs. To bring the ore to the reduction works gener-

[4] The tariff act of 1913 carried a 10 per cent. *ad valorem* duty. The present act, operative since October, 1922, imposes a tariff of 25 cents a pound on quicksilver, equal to $18.75 a flask, with corresponding duties on mercurials.

[5] Duschak, L. H., and Schuette, C. N., "Fume and Other Losses in Condensing Quicksilver from Furnace Gases," Tech. Paper 96, Bureau of Mines, 1918, 29 pp. Reprinted in *Min. and Sci. Press*, Vol. 117, 1918, p. 315.

ally costs two or three times as much as to treat it. This expense is due partly to the mode of occurrence of quicksilver deposits, and partly to the fact that mining operations are seldom conducted with maximum efficiency. Fluctuations in the price of, and demand for, quicksilver have tended to prevent operators from carrying on development in advance of actual mining; in consequence, the planning of a systematic mining program has been impossible. As an outcome of activities during the World War, however, a number of properties now have fair quantities of ore definitely blocked out. The authors believe that the greatest opportunity for increasing the economy in quicksilver production lies in giving more attention to the geology of the deposits and the improvement of mining methods. Further details are given in Bulletin 222 of the bureau.

The Science Exhibition at Wembley. (*Nature*, July 11, 1925.)—The key to the physical exhibits is a chart about 24 feet long, displaying 60 octaves of electromagnetic radiation. The methods of producing and detecting each range of wave-lengths are given. There is an overlapping of methods for wave-lengths of a few tenths of a mm., for these can be produced both by thermal and by electromagnetic means. "It is a remarkable fact that it is now possible to use a scale of wave-lengths as a guide to a very representative series of physical experiments: It emphasizes the change that has taken place in the orientation of scientific thought since the days when matter was everything and energy had not been defined, for now energy is paramount and matter is mentioned as an afterthought."

A succession of benches presenting illustrative apparatus and charts lead from the shortest to the longest wave-length. Gamma-rays furnish an opportunity to show the structure of the atom. In a piece of apparatus from the Clarendon Laboratory a single particle emitted by polonium breaks down the resistance of a spark gap with the result that the current thus caused actuates a loud speaker. In connection with ultra-violet radiation there is shown Sir Herbert Jackson's experiment "in which mixed visible and ultra-violet rays from a condensed aluminum spark are focussed by a quartz lens on a screen which fluoresces to ultra-violet rays of wave-length 1850 or 1860 Å.U. The visible rays are found to be focussed at about two feet from the lens and the ultra-violet of the above wave-lengths at about eight inches, so that by moving the screen it is possible to find two differently colored focal regions."

A famous experiment which, in the early part of the nineteenth century, had much to do with the victory of the wave theory of light over the corpuscular theory, is shown in a striking form, as "the

projection of the image of a luminous object by means of a spherical bicycle ball in place of a lens. Each point in the object throws a circular shadow of the ball having a white spot at its centre, and the aggregate of white spots forms the required image. Photographs can be reproduced by this method."

In the infra-red range of wave-lengths a spectrometer with a rock-salt prism is exhibited. By turning the prism all wave-lengths present that are in the interval from 5000 to 100,000 Å. can be made to fall in turn on a thermopile and thus announce their presence. By this means a Bunsen burner shows itself to emit strongly radiation of 44,000 Å.

Among geophysical apparatus is a seismograph that is so sensitive as to show the tilt of a coast when subject to the weight of a tide.

G. F. S.

Spiral Springs of Quartz. K. SLIUPAS and C. V. BOYS. (*Nature*, June 20, 1925.)—In connection with work on gravitation in the University of Lithuania, Kaunas, Professor Sliupas had need of a spiral spring of unusual delicacy and succeeded in his first attempt to make one from a thread of fused quartz. Three centigrams gave an extension of no less than a centimetre. The method employed is described. An oxy-acetylene flame was used to melt the quartz.

That prince of manipulators, C. V. Boys, tenders his congratulations upon the Lithuanian achievement, saying that in the late eighties of last century he tried to make a quartz spiral but failed. He describes a method by which he succeeded in producing a satisfactory spiral of glass thread.

In *Nature* (July 4, 1925), H. Greville Smith, of Bristol University, England, gives an account of his experience in making quartz springs. The Silica Syndicate has made a considerable number of such springs, having 15 to 30 coils from .5 to 1.5 cm. in diameter. The fibre was 1 or .2 mm. in diameter. Springs of these dimensions will support a total load of .8 gram and stretch 9 mm. upon the addition of .1 gram. They are thus more sensitive than the first spring of Professor Sliupas. He states, however, that this was greatly exceeded in delicacy by some of his later springs. Mr. H. D. H. Drane, in *Nature* (Aug. 29, 1925), gives details of a method of making springs from quartz fibres of about .001 cm. diameter. The fibre, originally straight, is wound about a quartz tube to which both of its ends are attached. Thus constrained it is heated for a short time at a temperature above the annealing point of fused quartz. For the diameter given above one minute at 1100° C. suffices. The coiled spiral then permanently assumes that form and keeps it after removal from the tube. By this process very sensitive springs can be prepared. One such spring was extended by its own weight at the rate of 30 km. per gram.

G. F. S.

THE FRANKLIN INSTITUTE.

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(Stated Meeting, Board of Managers, September 9, 1925.)

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LIBRARY NOTES.

RECENT ADDITIONS.

Annuaire Sucrier. Vols. 46–51. Six volumes. 1917–1925.

Beverage Blue Book, 1925. 1925.

BOOKER, JAMES P., and Others (Matthew Berman).—The How and Why of Candy Making. 1925.

Faraday Society.—The Physical Chemistry of Igneous Rocks: A General Discussion Held by the Faraday Society, the Geological Society, and the Mineralogical Society. 1924.

LAUE, M. VON, and Others.—Die Theorien der Radiologie. Vol. 6. 1924.

LECAT, MAURICE.—Bibliographie de la Relativité. 1924.

MATAGRIN, A.—L'Industrie des Produits Chimiques et ses Travailleurs. 1925.
MONCKTON, T. A., Compiler.—Hydrogen Manual. Vol. 2. 1920.
Ohio Miami Conservancy District. Technical Reports. Vol. 10. 1925.
REYNAUD-BONIN, E.—Appareils et Installations Téléphoniques. 1924.
TOWLES, WILL H.—Towles' Portrait Lightings. 1925.
West Virginia Geological Survey. County Reports, 1924: Mineral and Grant Counties. Two volumes, text and maps. 1923–1924.
ZSIGMONDY, RICHARD.—Kolloidchemie, pt. 1. 1925.

BOOK REVIEWS.

APPAREILS ET INSTALLATIONS TÉLÉPHONIQUES. Par E. Reynaud-Bonin, Ingénieur en chef des Postes et Télégraphes, Professeur à l'École supérieure des Postes, Télégraphes et Téléphones. 487 pages, 6″× 9″, illustrated. Paris, Librairie J.-B. Baillière et Fils, 1924. Price, paper, 50 francs; bound, 60 francs.

In general principle, the telephone of fifty years ago is still recognizable. Progress in the art has been by slow degrees, persistent refinement of detail, a stroke of genius here and there and unremitting effort in meeting the progressive exactions of the user and overcoming the difficulties of long-distance transmission. While of the multiplicity of methods and devices which have been proposed, relatively only a few have survived the test of performance in actual service, they nevertheless constitute a formidable collection. It is these appliances and circuits in current use with reference to matters of historic interest which the author describes, first mainly in a descriptive manner and further on analytically.

The work is in seven main divisions. Book I contains a historic review, a general description of receiving and transmitting apparatus and the equipment of a subscriber's installation. Book II describes manually-operated central stations. In Book III inter-communication between central stations and a general survey of the line network are considered. Book IV deals with private exchanges. The various developments of the automatic telephone are detailed in Book V. Book VI covers a mathematical analysis of telephone apparatus. Book VII is on the theory of telephone lines. A bibliography and indexes of names and subjects are included. The work is one of an extensive series on a wide variety of industrial engineering subjects and is issued under section III of this series, "Encyclopédie d'Électricité Industrielle," of which thirty-two other volumes have appeared.

The subject-matter is presented in an unusually clear manner, somewhat suggestive of the correspondence school texts of this locality. It is illustrated by about 300 well-executed cuts and diagrams of circuits. As is to be expected, in the main, European practice is described, though American practice is by no means ignored, as is evidenced by the frequent references to the contributions of the Western Electric Company. The theoretical discussions of Books VI and VII are thoroughly modern. Lengthy abstracts of the work of Poincairé, Kennelly and others on the telephone receiver are representative of the material of these sections. There is a chapter on the non-electromagnetic receiver and

other topics of like character. Some historic notes and an introduction to the mathematical processes employed in the analysis of telephone lines are contained in the final section. Needless to say the work of Pupin and the application of electronic amplifiers receive adequate attention.

The work is comprehensive and with its notes and comments highly interesting.

LUCIEN E. PICOLET.

TOWLES' PORTRAIT LIGHTINGS. A practical method for making portrait lightings. By Will H. Towles. 103 pages, including forty-four illustrative photogravures, a portrait of the author and of Daguerre. Folio form. Philadelphia, Frank V. Chambers, 1925. Price, $5.

The author of this work is director of a school of photography. His portraits show that he is master in the field. Obviously, the photogravures, though executed in the highest style of the art, do not do full justice to the originals. Photography, other than the purely scientific applications of it, such as photomicrography, requires skill in lighting and posing, but also considerable of the artistic sense. Few persons care for a merely map-like counterfeit presentment. Fads, of course, arise and pass off, as in other human affairs. The older members of the profession will recall the "carte de visite," the Rembrant lighting, the cabinet form and others of less importance. To-day the "fuzzigraph" is much in evidence, dignified in the literature as the "soft focus." Probably an excellent portrait could be obtained by the pin-hole method if some research worker would produce an extremely rapid emulsion so that the exposure would require only a fraction of a second. The psychologic factor enters into portrait work, and it not infrequently happens that the "push the button and let the company do the rest" camera fiend gets excellent pictures of his friends because they are entirely at ease before him. It is, of course, known to all who do general photography that the eye may overlook conditions that the camera will emphasize so as to give a disappointing picture. Strolling about in search of pretty landscape effects, the amateur may snap a view and on developing it find a deep deposit of silver appearing unexpectedly, and turning out to be wash hanging in a yard, but overlooked by him when setting the camera. Even skilled studio workers may overlook an awkward detail. In the portrait of the old man (illustration eight) the mouth has been allowed to remain open, giving to the picture a peculiar stupid expression suggestive of mouth breathing. No doubt this was the normal condition of the subject, but the studio slogan "look pleasant, please," should have been in modified form addressed to the subject in this case. In present-day work it has become quite common to photograph young women, full or three-quarter face, with lips parted showing teeth. It is a fashion, and it is not for the reviewer, who has no real studio experience, to condemn it, but most men will agree that women are more to be admired when they keep their mouths closed.

Professional photographers will find in this book admirable and abundant information concerning the arrangements of the studio for securing efficient lighting. The book itself is a fine example of the excellent printing and general make-up that characterize the Chambers press.

HENRY LEFFMANN.

THE BEVERAGE BLUE BOOK FOR 1925. 336 pages, 8vo. Chicago, H. S. Rich and Company.

The "soft drink" industry has reached enormous proportions in America. Its development has been largely due to the heat of the American summer, but the tendency to avoid intoxicating beverages has probably had a noticeable influence. The book in hand shows vividly the present condition of the trade, the development of which has given rise to numerous accessory industries that exemplify American ingenuity and efficiency in mechanical invention. It would be well if the same characteristics could be manifested in some other departments of applied science.

At the present time the entire process of making bottled beverages is carried out by machinery without contact by human hands. It is very interesting to note in the advertisements, the ingenious machinery for washing, filling, capping and labelling bottles. Bottles themselves are now made by machinery, but this phase does not concern the bottling trade. "Soda-water" was originally merely a tumbler of "fizz," obtained by mixing, off hand, salæratus with an acid salt, but the name has been retained, although the effervescence is now due only to the escaping carbon dioxide. While the greater portion of the book is devoted to a list of the firms engaged in the manufacture of bottled beverages, a notable amount of space is devoted to scientific articles, among which may be noted "water supply," "causes of deterioration in carbonated beverages," "weather forecasting." The last title may awaken curiosity. It concerns the probable temperature of the season in which the soft drinks are mostly used. The bottler would be glad to know whether the summer which he awaits is to be cool or hot, for upon the given condition depends the volume of his sales. The official weather bureaus have been of late years giving limited prognostications, but have refused to be drawn into any long-range predictions.

There are many interesting and valuable data in the book. There is one phase of the beverage industry that has not yet received the attention it should, namely, the washing of the glasses in which the drink is served. In many places the glass or mug is simply rinsed in a small tub. The introduction of paper cup and straws has been an improvement, but there is still much reform to be accomplished in securing cleanliness in this service.

HENRY LEFFMANN.

HANDBUCH DER RADIOLOGIE. VI. Die Theorien der Radiologie. By v. Laue, Zeeman, Lorentz, Sommerfeld, Wentzel, Joos, the late E. Riecke, Vegard and Debye. Edited by Erich Marx. xi–806 pages, 8vo, paper bound. Published by the Akademische Verlagsgesellschaft m.b.H., Leipzig.

This is the closing volume of a work of most extensive and thorough character in a field of physics of comparatively recent rise to importance. The term "radiology" suggests nowadays several phases. It may refer to the phenomena of radio-activity, to the X-ray and its manifestations or to the wireless transmission of sounds, now so popular under the name "radio." As the reviewer has only knowledge of the volume in hand, it is impossible to say how far the remainder of the work entered into the several phases above enumerated, but inasmuch as they are all phenomena of radiation, it may be presumed that they have been fully considered. Also as the volume in hand is

devoted mostly to the theories of radiant phenomena, it may be assumed that much practical information was given in the earlier volumes.

It will be seen by the abstract of title given above that the subject-matter has been prepared by some of the most distinguished authorities in this department of physics, and we may be sure that the statements are comprehensive and correct. The opinion of physicists seems to be coming round to the Newtonian view of the nature of radiation, the vibratory theory which dominated the science for so many years having ceased to be so earnestly urged. It is an interesting question as to what will be done with the many technical terms which have been constructed on the basis of the undulatory theory.

About ninety pages are devoted to the nature and phenomena of the aurora. This striking and often highly impressive manifestation of radiant energy is seen comparatively frequently in high latitudes, rather rarely in temperate regions and not in the equatorial. Strictly speaking, it should be called the polar light, for it is manifested in the southern hemisphere also. That we call it usually the *aurora borealis*, or northern lights, is merely one of many imperfect designations of the phenomena of nature, that have arisen from the fact that the dominant civilization has been developed in the regions north of the equator. Many of our festivals are seasonal in origin, depending on the harvests, which are dependent on the course of the sun. Early man celebrated the winter solstice (rebirth of the sun) by feasting, and similarly enjoyed the autumn ingathering. When Europeans began to colonize the southern hemisphere they experienced a peculiar upset in their "appointed days." In South Africa, New Zealand and Australia, spring poetry is in order in September; May finds the trees in most cases in sere and yellow leaf. Overcoats are in order in July, open-air sports in December.

The variation of the aurora with latitude was first studied by Loomis in 1860. He pointed out that the increase of frequency was not coincident with increase of latitude but that it occurred in two zones about 20° from the two so-called northlight poles.

The article in the volume in hand is by Vegard, who points out the great variability of the phenomenon in location, form, color and intensity. That it has a close connection with electrical manifestations was forcibly noted after land telegraph lines were established, for these were often operated without battery supply during the appearance of a marked light. Long previous to such observations, the coincidence of auroral phenomena with so-called "magnetic storms" was noted. Halley, in 1716, and Hiorter and Celsius, in 1741, gave attention to the relation between the two, and it is now established that they are most marked in the zone of the aurora. Vegard, in 1912–1913, in an expedition to northern regions, studied the condition. Now that the radiant energy, commonly called light, is regarded as an electromagnetic phenomenon, the influence of an aurora on aerial wires is not astonishing. It would be interesting to know if any observation has been made on the effect of a well-developed aurora on broadcasting. Perhaps there would be a great deal of static.

Vegard divides the forms of the aurora into two groups, quiet and active, and each of these is further subdivided. In connection with the description, a page of photogravures is given, illustrating appearances of the light, also plates

of spectra thereof. The spectra were taken through a quartz optical train, but the photographs of the light itself were taken in the years between 1910 and 1914, both inclusive, and were, therefore, taken probably through the ordinary photographic lenses and on standard plates not treated for panchromatism. As all the pictures are in monotone, the really imposing appearance of the light is not reproduced. Vegard made extensive spectrographic studies with quartz and glass apparatus, and obtained very striking and valuable results, concerning the chemical composition of the upper atmosphere. It has usually been considered, at least of late years, that in the upper-air regions hydrogen and helium are abundantly present, in fact, are practically the whole mass. Vegard's observations negative this, and he interprets his results as indicating solid nitrogen in finely divided condition. He applies this inference to an explanation of the zodiacal light and to the twinkling of the fixed stars. His results have been obtained by most careful studies of the natural phenomena and elaborate experimentation, so that he feels authorized to speak positively on the subject. Other workers have not accepted his inferences entirely. Among other points it has been claimed that the lines observed are due to oxygen, but this Vegard combats earnestly. He also expresses strongly the opinion that the spectroscopy of the aurora does not justify the belief in an undescribed element in the upper air, which has been posited by some under the provisional title of "geocoronium." After all, there is a great deal of uncertainty in the inferences from observations on distant regions and scientists should take to heart the recent utterance of Millikan to be very cautious in stating their views and avoid dogmatism. To many persons, indeed, these discussions of far-away conditions bring to mind the advice that was given by the one whom Mr. Micawber called "the immortal exciseman beyond the Tweed,"

> "To go up in balloons,
> And stay a month amang the moons
> And see 'em right."

Seriously, however, the work in hand offers a most comprehensive and careful presentation of the theories now current in the domain of radiology and will afford to the advanced student and research worker in the subject immense aid. The general execution of the book is good. Scientists of other countries than Germany may feel thankful that their German brethren have chosen to print all scientific literature in Roman characters and not in the national text, the product, as Whitney remarked, of the perverse ingenuity of the monks of the middle ages. The frequent paragraphing common in books of continental writers appears in this, and unnecessarily increases the space occupied by the text. An index is given, but it is for the volume only. It would seem that the closing volume should have a general index. HENRY LEFFMANN.

KOLLOIDCHEMIE. Ein Lehrbuch. By Dr. Richard Zsigmondy, University of Göttingen. Fifth edition, revised and enlarged. Vol. i, General Part. xiii–246 pages, 8vo, paper bound. Leipzig, Otto Spamer. Price, 11 marks.

The first edition of this work appeared in 1912. Since then events of most tragic nature and profound influence have occurred. Germany has shown the world the value of the coöperation of theory and practice in science, a lesson

that has not gone wholly unappreciated, but it is a question whether the peoples who constituted what were designated in the war "the allied and associated powers" have learned the lesson as thoroughly as they should. That five editions should have appeared in the space of thirteen years, including more than four years of the most intense war that the world has witnessed, shows the merit of the book, yet we must give due credit to the standing of the author in this field, which in many ways he has made his own.

The present edition has been carefully revised and is more extensive than its predecessors. In the initial chapter a very earnest effort is made to set forth the exact nature of colloids, yet the subject is at best abstruse. Colloid chemistry has found extensive and remarkable applications, and the literature devoted to it has reached an enormous volume. Zsigmondy begins the treatise with a short historical notice in which, of course, Graham figures as the initiator by his comprehensive and ingenious researches. In this, as in so many other cases, the British initiative was not immediately fertile and the exploitation of the field passed to the Germans, whose labors have greatly expanded the pure and applied phases of the phenomena and whose books and journals have embodied the greatest amount of collated data.

As the volume in hand is the fifth edition and as the standing of the author is of the first rank, it is not necessary to speak at length of the merits of the work. It has been so long before the public and so highly appreciated that it needs no special encomiums. The general subject of colloid chemistry is set forth in sufficient detail for all who desire a knowledge of its principles or a guide to further research.

HENRY LEFFMANN.

THE STORY OF SUGAR. By C. F. Bardorf, Canadian Institute of Chemistry. vii–191 pages, 12mo. Easton, Penna., Chemical Publishing Company, 1924. Price, $1.50.

This is a popular treatise on the source and manufacture of common sugar with considerable information as to its chemistry and relationships. The style is familiar and lively. Sugar is an important item in American life. The United States consumes yearly almost one hundred pounds for every person, and takes, therefore, almost one-quarter of the world's production. The crisis in sugar supply that occurred in the latter years of the great war and immediately thereafter is still vivid in the memories of the great nations. The relation of tobacco and sugar to the development of negro slavery in America is pointed out. Information is given as to the earliest allusions to sugar and the gradual spread of its use over the civilized world. The sources from the cane and the beet are mentioned. It appears that the crusaders found the cane under the name of "zukra" in Syria, from which the European languages have evolved the terms for the substance. Even Esperanto (sukero) is not overlooked, the word following the invariable Esperanto rule that all nouns end in "o." There is a very large amount of valuable and useful information to be found in the book; an autobiographical style has been adopted, that is, sugar is assumed to tell its story. The selection of this is a matter in which an author has a free choice. The reviewer does not admire it, but that is a minor point. There is no index, but a comprehensive table of contents.

HENRY LEFFMANN.

THE ELEMENTS OF SUGAR REFINING. By C. F. Bardorf and J. A. B. Ball. x–240 pages, 8vo, illustrated. Easton, Penna., Chemical Publishing Company, 1925. Price, $4.

The authors of this book are fellows of the Canadian Institute of Chemistry. The senior author has treated the story of sugar in a popular form, notice of which will be found above. The work in hand is a formal presentation of the technology of sugar refining. Notwithstanding the immense literature on the subject, the authors state that the only other recent available work on the subject is in German, which is almost entirely devoted to European methods, and hence the need for a work covering American practice. The sugar industry is one of great extent and importance. It seems that in comparison with some other great industries, the problems presented are not very difficult, yet there is considerable opportunity for improvement in both mechanical and chemical procedures.

Sugar is used very largely at the present day, probably much more than is best for general human health. It stands on the boundary between true foods and accessories. There seems to be no doubt that it contributes to the body-heat, but it is not quite as digestible as is commonly supposed. Unlike true foods, it leads to excess. A true food satiates when a reasonable amount of it has been consumed, but sugar, appealing to the sense of taste, provokes further use when enough has been taken and thus overbalances the ration. In this respect sugar associates itself with alcoholic beverages, and the milder drinks such as coffee, tea and cacao. The United States, as is well known, is the greatest *per capita* consumer of sugar, and this use is largely due to the popularity of candies and soft drinks. The improvements in sugar manufacture in the last half-century have been considerable. The cruder methods of extraction of the syrup and its concentration led to much loss of crystallizable material. The resulting molasses usually contained in suspension a notable amount of sugar. Fifty years ago commercial molasses would, on standing, yield a deposit of sugar, but the application of the centrifuge changed this.

The book under notice covers the details of American practice in dealing with sugar from the raw material to the marketable article. The several processes are described fully, and, where appropriate, illustrations are given. In addition to purely mechanical details, the apparatus used for assaying the raw material and determining colors are given. The Lovibond tintometer is recommended in a form modified by the authors, which has been found very satisfactory. The relative merits of the Lovibond apparatus and the Hess-Ives are not discussed, as the journalistic literature on sugar technology has presented that question fully. Much information is given as to general testing procedures. Diagrams and tables are used freely, and the more important accounting problems are treated clearly. The book, therefore, is a comprehensive treatise on the entire phase of a great industry. The literary style is very defective. Little attention seems to have been paid to correct form. Many sentences are turgid and awkward. Neglect of ordinary grammatical rules is occasionally to be noted. Such expressions as "silicious skeletons of a minute aquatic plant known as diatoms" and "errata slip" when there is only one erratum should, perhaps, have been caught by the proof-reader. This laxity of literary form is becoming more and more evident in American scientific works, and the "well of English

undefyled" that Spenser praised is no longer sufficiently appreciated. The price seems a little high in view of the size, but printing costs have increased enormously in this country of late years and publishers are at the mercy of the composing room. The invention of type-setting machines seems to have increased the cost of composition. HENRY LEFFMANN.

ASTRONOMIC DETERMINATIONS BY THE UNITED STATES COAST AND GEODETIC SURVEY AND OTHER ORGANIZATIONS. By Sarah Beall, Mathematician, Coast and Geodetic Survey. 337 pages, 25 illustrations, quarto. U. S. Coast and Geodetic Survey Special Publication No. 110.

Astronomers, engineers, teachers, and scientists generally will welcome the appearance of Special Publication No. 110 of the United States Coast and Geodetic Survey, entitled "Astronomic Determinations by the United States Coast and Geodetic Survey and Other Organizations," a volume of 337 quarto pages, well illustrated and fully indexed, containing some 766 determinations of latitude, 378 of longitude, and 616 of azimuth, together with certain related data.

In her introduction to this volume, the author makes the following statements:

> "The purposes of the geodetic operations of the Coast and Geodetic Survey are, first, the control of the charts published by the Survey; second, the furnishing of geographic positions (latitudes and longitudes), accurately determined elevations, distances, and azimuths to field parties of this bureau and to other organizations; third, the determination of the shape and size of the earth.
>
> "In order that points of a survey may be accurately located on the face of the earth by their coördinates of latitude and longitude, the astronomic latitude and longitude of occasional stations of the Survey must be determined and also the astronomic azimuths of some of the lines. Accurate astronomic determinations have therefore been an important part of the geodetic operations of the Coast and Geodetic Survey, and men of the highest scientific training have been brought to the Survey to aid in the development of this work."

All astronomic data determined by the Coast and Geodetic Survey during a period of seventy-five years have been gathered together in this volume, much of it here appearing in print for the first time. Among the other organizations whose astronomic determinations are included are the United States Lake Survey, the United States Geological Survey, the United States General Land Office, the United States and Canada Boundary Commission, and the Corps of Engineers, United States Army.

The data are given in carefully prepared tables, designed to show not only the final results of the determinations, but all critical data pertinent thereto. There are also given descriptions of the topographic locations of the points to which the data pertain, and of their station marks.

But this volume is more than a compendium of astronomic data. In it will be found brief but well-illustrated descriptions of the methods and instruments used in the field, and a statement relating to the effect of variation of the pole, while in Chapter IV will be found a clear statement concerning the deflection

of the vertical, together with a tabulation of the deflection at the various stations. In this chapter also appears a definition of the standard geodetic datum used in the horizontal control surveys of the United States, and of her near neighbors, Canada and Mexico.

This volume should prove a most useful reference book to all students of geology and of geography, as well as to those already mentioned above, while in college and university libraries it will fit snugly into a long-vacant niche.

HUGH C. MITCHELL.

ELECTRICAL PRECIPITATION. A Lecture delivered before the Institute of Physics, by Sir Oliver Lodge, D.Sc., F.R.S. 49 pages, 5 plates, 8vo. Oxford University Press, American Branch, New York, 1925. Price, $1.

This is Volume 3 of a series of essays on physics in industry, a series appearing under the auspices of the British Institute of Physics, which is also the publisher of the *Journal of Scientific Instruments*. The great benefit both to some industrial methods and to the public at large that has resulted from the introduction of artificial methods of precipitation is now well known. Dust is the settled, or possibly in a spirit of punning, we might say the "unsettled" item of general discomfort. Older people are generally inclined to talk about the degeneracy of the times in comparison with conditions when they were young, but while much of this may be considered as due to the effect of perspective, there is no doubt that in the large eastern cities of the United States, a sad pollution of the atmosphere has resulted from the enormous industrial development, especially brought about by the increasing cost of hard coal and the thereby increasing uses of the softer, smoky varieties.

Sir Oliver Lodge's essay discusses natural and artificial precipitation and the combination of the two with suggested meteorologic possibilities. An appendix treats of practical phases of commercial precipitation. The "urge" to apply these methods, especially the artificial, will be the possible profit from recovery of the dust and the protest of the community against the pollution of the atmosphere. There is now no doubt that the introduction of dust into the air brings about unfavorable conditions other than the direct injurious action. The section on natural precipitation gives many data about meteorologic phenomena, among others a study of the nature of thunderstorms and allusion to the aurora. The physics of Dr. G. C. Simpson's theory of thunderstorms is given in brief but comprehensive form, and will be interesting reading to all who are studying the physics of the atmosphere.

People at large will, however, be much more interested in the section on artificial precipitation. Noting the early history, we find that the author gives us an example of how difficult it is to determine the originator of a method. He states that in 1884, in association with Mr. J. W. Clark, he made at Liverpool some observations on artificial precipitation, and found later that an article by Mr. Guitard had appeared in *The English Mechanic* in 1850, but even this observation was anticipated by a Mr. Hohlfeld, of Leipzig, as early as 1824. Practical application of the function of certain electrical charges to cause prompt and complete settlement of many forms of dust have been in use for some time. British and American experts have entered into amicable business relations. The Lodge Fume Deposit Company and Doctor Cottrell,

the American inventor, who has done such good service in this field, have joined hands. A German company employing patents of German origin has also agreed to operate without antagonism to the British-American company, the agreement being concisely expressed by one of the now favorite diplomatic phrases "delimitation of frontiers."

The essay is illustrated with five fine plates. HENRY LEFFMANN.

THE PHYSICAL CHEMISTRY OF IGNEOUS ROCK-FORMATION. A General Discussion Held by the Faraday Society, the Geological Society and the Mineralogical Society. Reprinted from the *Transactions of the Faraday Society* (1925, 20). Discussion held October 22, 1924. 88 pages, 8vo. Published by the Faraday Society. Price, 6s.6d.

Without intending a play on words, but merely as an accepted form of expression, it may be said that mineralogy is in the last analysis a department of chemistry, and as chemistry is steadily being absorbed by physics, one is not astonished that the physical chemistry of rock-formation is beginning to attract attention. Mineralogists have, indeed, often relegated chemistry to a subsidiary place in their writings. They have, of course, not ignored chemical composition, but they have laid especial stress on form. Crystallography has figured very prominently in text-books on mineralogy; in fact, in modern times, no one can become an efficient worker in the subject without a thorough knowledge of the mathematics of crystallography. The introduction of the sectioning of minerals so as to permit of microscopic examination added a most important and fertile method of research and gave rapid rise to petrology, which became very useful in geology. Mankind has always been interested in minerals, both from purely economic as well as sentimental reasons. Ores and gems have been sought in all parts of the world. With the rise of quantitative analysis, and the development of methods of securing high temperatures, experiments on the production of well-known rock constituents were undertaken actively. Doctor Flett, who delivered the introductory address, gave an account of the earlier work in this field. It was not until the beginning of the latter half of the last century that material advance was made.

The introduction of physical chemistry into the study of rock-materials has resulted from the increase of knowledge of the nature of solution, which has been industriously cultivated in recent years. It appears that considerable valuable information has been obtained by the study of fusions under simultaneous use of steam. Credit is given to the Geophysical Laboratory of the Carnegie Institution of Washington and its Director, Doctor Day, for especial service in the field. Every precaution has been taken to secure accuracy and certainty. The work of the laboratory is summarized. There has been a great tendency of recent years to follow this work, and continental universities have entered enthusiastically into further research. In the course of the discussion Mr. Cosmo Johns, of Sheffield, expressed the view that British scientists cannot expect any such development promptly in their own land, but alternative methods that will have great value were mentioned. Interesting data concerning glass manufacture were brought out, and it was suggested that the manufacturers thereof might be able to afford excellent opportunities for experiment.

HENRY LEFFMANN.

PUBLICATIONS RECEIVED.

Physiological and Clinical Chemistry, by William A. Pearson and Joseph S. Hepburn. 306 pages, interleaved, illustrations, 12mo. Philadelphia, Lea and Febiger, 1925. Price, $4.

Chemistry in Modern Life, by Svante August Arrhenius. Translated from the Swedish and revised by Clifford Shattuck Leonard. 286 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1925. Price, $3.

The Metallurgy of Aluminum and Aluminum Alloys, by Robert J. Anderson. 913 pages, illustrations, 8vo. New York, Henry Carey Baird and Company, Inc., 1925. Price, $10.

The Story of Sugar, by C. F. Bardorf. vii–191 pages, illustrations, 12mo. Easton, Chemical Publishing Company, 1924. Price, $1.50.

The Elements of Sugar Refining, by C. F. Bardorf and J. A. B. Ball. x–240 pages, illustrations, 8vo. Easton, Chemical Publishing Company, 1925. Price, $4.

Hydraulics and Its Applications, by A. H. Gibson. Third edition, xv–801 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1925. Price, $6.

Steam Condensing Plant in Theory and Practice, by James Sim. xiii–271 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1925. Price, $6.50.

Introduction to Qualitative Organic Analysis, by Hermann Standinger. xvi–112 pages, 8vo. New York, D. Van Nostrand Company, 1925. Price, $2.50.

Electrical Precipitation, by Sir Oliver Lodge. 49 pages, plates, 8vo. London, Oxford University Press, 1925. Price, $1.

The Homo Method for the Drawing of Steel. Catalogue No. 93. 19 pages, illustrations, quarto. Philadelphia, Leeds and Northrup Company, 1925.

National Advisory Committee for Aeronautics: Technical Notes No. 223, Determination of the Lift and Drag Characteristics of an Airplane in Flight, by Maurice W. Green. 16 pages, plates. No. 224, Pressure Distribution on the Nose of an Airship in Circling Flight, by Karl J. Fairbanks. 4 pages, plate. No. 225, Propeller Scale Effect and Body Interference, by Fred E. Weick. 7 pages, plates, table, quarto. Washington, Committee, 1925.

Ontario Department of Mines: Thirty-third Annual Report, being Vol. xxxiii, Part VI, 1924. 52 pages, illustrations, maps, 8vo. Toronto, 1925.

United States Department of Commerce: Technologic Paper of the Bureau of Standards, No. 287, A Hot-wire Anemometer for Measuring Air-flow through Engine Radiators, by Carl G. F. Zobel and L. B. Carroll. 12 pages, illustrations, diagrams, quarto. No. 288, Comparative Cold-rolling Tests of Open-hearth steel Strip (Deep-drawing Stock) and Electrolytic Iron Strip, by John R. Freeman and R. D. France. 20 pages, plates, diagrams, quarto. No. 288, Comparative Slow Bend and Impact Notched Bar Tests of Some Metals, by S. N. Petrenko. 34 pages, illustrations, plates, diagrams, quarto. Circular No. 276, Motor-vehicle Headlighting. 28 pages, illustrations, plates, quarto. Washington, Bureau of Standards, 1925.

United States Department of Commerce: Elimination of Waste: Simpli-

fied Practice Recommendation, No. 36, Milling Cutters. 25 pages, tables, 8vo. Washington, Government Printing Office, 1925.

United States Department of Agriculture: Department Bulletin No. 1346, Status of the Pronghorned Antelope, 1922–1924. 64 pages, illustrations, plates, maps, 8vo. Washington, Government Printing Office, 1925.

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